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16. ABSTRACT

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49th Annual Meeting of the
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A review of laboratory test data reveals that most common highway materials, under conditions representative of moving traffic on an in-service pavement, exhibit a non-linear response to stress. The reported stress-strain response of pavements constructed with such materials varies from the stress-softening to the stress-stiffening type, in accordance with the response of the constituent materials. A non-linear elastic incremental finite element analysis of a uniform sand mass subjected to a uniform circular surface load, using a constitutive equation based on published laboratory data, revealed a pronouncedly stiffening relation between the applied pressure and surface deflection, and slightly non-linear relations between the applied pressure and the vertical stresses induced in the mass. An approximate non-linear elastic analysis of a full-depth asphalt concrete pavement over a sandy clay subgrade, using stress-strain coefficient matrices measured in laboratory triaxial tests on the materials, gave almost linear relations between the applied pressure and the resulting deflection, and distributions of stresses and strains within the structure very similar to those yielded by a linear elastic analysis using stress-strain coefficients at realistic stress levels. To an engineering approximation, a linear analysis was sufficiently accurate in the case of this particular full-depth asphalt concrete pavement, but appeared unacceptable in the case of a pavement with unbound granular materials close to the surface.

INTRODUCTION

Analyses of stresses and strains induced in pavements by traffic form part of many pavement design and evaluation procedures, including some which are primarily empirical, and those which are primarily mechanistic. In almost all cases these analyses are based on linear elastic theory, implying important assumptions regarding the stress-strain-time response of the component materials.

The response of common highway materials under stress typically includes elastic, viscous, and plastic components. During the first cycle of stress, at slow loading rates, or at high stresses, the viscous and plastic components may be dominant. For conditions representative of moving traffic on a well-designed in-service pavement (many short-duration repeated stresses of limited intensity) the strains are largely elastic, and only small permanent strains result from a single vehicle passage. This "resilient" response is dependent upon stress intensity, in that most highway materials exhibit non-linear stress-strain relations (1-14). In this paper observations of non-linearity in the response of pavement structures to load are reviewed, and analyses of stresses and strains induced by traffic in pavements constructed of non-linear elastic materials are described, in an effort to obtain an indication of the degree of error resulting from the common simplifying assumption of linear elastic behavior.

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NON-LINEAR RESPONSE OF HIGHWAY MATERIALS TO STRESS

Before proceeding to the behavior of pavement structures, non-linearities in the resilient response of common highway materials under conditions representative of in-service pavements will be reviewed briefly. Most laboratory work has been carried out in triaxial apparatus by applying short-duration cyclic stresses of intensities corresponding to those induced in a pavement. Usually only the axial stress has been varied, the confining pressure being kept constant. The results have been expressed as a resilient modulus[1] which varies with stress level.

The resilient modulus of unbound sands and gravels has been observed to increase with the confining pressure or mean normal stress, and to be essentially independent of the magnitude of the repeated deviator stress until shear failure approaches, when it decreases rapidly (e.g. Biarez (1), Trollope et al (2), Morgan (3), Dunlap (4), Seed et al (5), Kallas et al (6)). Several experimental investigations have shown that the resilient modulus (M_R) may be expressed by the relation

$$M_R = K_1 (I_1)^{K_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where K_1 is a constant, I_1 the sum of the three principal stresses, and K_2 an exponent varying from 0.35 to 0.6; or more simply by

$$M_R = K_3 (\sigma_3)^{K_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where σ_3 is the confining pressure, K_3 a constant, and K_4 varies from 0.35 to 0.55. Similar relations have been established theoretically, for example by Ko and Scott (7).

All studies of the non-linear response of clays have indicated that the resilient modulus in the direction of a given stress decreases as that stress is increased, and

[1] Resilient modulus is a secant modulus obtained by dividing the change in axial stress by the recoverable axial strain.

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is little affected by the transverse stresses (Seed et al (8), Kallas et al (6), Brown et al (9)). In tests on a silty clay, for example, Seed et al observed a 400 percent variation of resilient modulus between repeated stresses of 3 and 15 psi.

The modulus of a lime-treated clay has been observed to increase with increasing effective confining pressure, and decrease with increasing deviator stress (Fossberg (10)). The moduli of some cement-treated sands and clays have been found to decrease with increasing deviator stress and, in some cases, to increase with confining pressure (Mitchell et al (11), Wang (12)). Wang found that the stress dependency of a cement stabilized silty clay could be expressed by the relation

$$M_R = K_5 \cdot (K_6 - \log_e \sigma_d)\, I_1^{K_7} \qquad (3)$$

where K_5, K_6 and K_7 were constants, σ_d the repeated stress difference, and I_1 the sum of the normal stresses. Despite this non-linear response in triaxial tests, the cement treated materials appeared to behave linearly in beam flexure.

The resilient modulus of a sand-asphalt mixture has been observed to increase markedly with an increase in cell pressure or a decrease in deviator stress (Gregg et al (13)). Asphalt emulsion treated aggregate has shown a pronounced increase in resilient modulus with increasing confining pressure soon after compaction, but this dependency became less marked at long curing times when the emulsion had broken and a strong asphalt-aggregate bond developed. The modulus decreased with increasing deviator stress at all ages. A mixture of a straight-run asphalt with aggregate showed a slight increase in modulus with increasing confining pressure, and a more marked reduction with increasing deviator stress (Terrel (14)).

Most of these investigations have been limited to tests in which uniaxial repeated stresses were applied. These are barely representative of conditions in a pavement, where changes in three normal and three shear stresses occur simultaneously. Similarly, most work has been concentrated on the response in the direction of the

applied stress, and little knowledge exists as to the response in the traverse directions (i.e. the Poisson effects).

In view of the non-linear stress-strain response, and the essentially recoverable nature of the strains after a large number of repetitions of short-duration stresses, a non-linear elastic model[2] appeared potentially suitable to represent the resilient[3] response of asphalt concrete, clay, and sand for conditions of moving traffic on an in-service pavement. To completely characterize a non-linear elastic material it would be necessary to measure all six strains induced by all possible combinations of six normal and shear stresses. As a step in this direction triaxial tests on undisturbed samples of silty clay and asphalt concrete were conducted (16) in which measurements were made of axial and radial strains resulting from repeated axial and radial stresses,[4] varied independently over a range of stress intensities and stress combinations which spanned those experienced by in-situ materials under traffic. The results obtained, expressed as incremental stress-strain coefficient matrices at various reference stress states, have been discussed fully elsewhere (16), and for present purposes will only be summarized as simply as is possible here. An example of results obtained in tests on an asphalt concrete is shown in Fig. 1, where the axial strains resulting from simultaneous applications of axial and radial stresses of various magnitudes have been plotted three-dimensionally. From this figure a slightly

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[2] A non-linear viscoelastic model might be more appropriate, particularly in the case of asphaltic material, but the complexity of experimental techniques and boundary value problem solutions for such a model discourage any attempt to apply it to the materials forming a pavement structure at this stage. Dong, Pister, and Dunham (15) have however formulated experimental and analytical procedures for a non-linear viscoelastic model and have applied them to a simple problem.

[3] It must be emphasized that this model can only be hoped to simulate the resilient aspect of the material response, and is inadequate for time-dependent strains under long-term stresses, or cumulative permanent strains after repeated applications of short-duration stresses.

[4] Stresses were applied for 0.1 second every 3 seconds.

stiffening non-linear response under increasing applied compressive stresses, and a more markedly softening response under increasing tensile stresses, is apparent. Results of another test, plotted two-dimensionally, are shown in Fig. 2, and by comparison with those in Fig. 1 reveal that an increase in the temperature of test from 70 to 94^{o}F resulted in an increased degree of non-linearity of the stress-strain relations. In all tests on silty clay samples the strains in the direction of, and transverse to, an applied stress increased more rapidly than did the stress itself (indicating non-linearity of the stress-softening type), but were little affected by the magnitudes of the transverse reference stresses. Expressing this differently, the resilient modulus (the applied stress divided by the resulting strain in the same direction) decreased with increasing applied stress, independently of the transverse stresses, as reported previously for clays by others (6, 8, 9). In both materials the Poisson coefficients (ratio between the strains transverse to, and those in the direction of, an applied stress) remained approximately constant or increased slightly with increasing applied compressive stress, and were essentially independent of the magnitude of the transverse stress. Both materials were initially cross-isotropic, with the horizontal stiffnesses exceeding the vertical. Significant degrees of stress-induced cross-isotropy were also observed, which varied with the magnitudes of the reference axial and radial stresses.

OBSERVATIONS OF NON-LINEAR RESPONSE OF PAVEMENT STRUCTURES TO LOAD

Measurements of deflections, stresses, and strains in pavement structures subjected to realistic repeated loads through plates have revealed non-linear response of the structures, and some examples follow. In plate tests on plastic clay subgrades, stress-softening load-deflection relations (i.e. the apparent modulus of clay decreasing with increasing plate pressure) have been reported by several investigators (Seed et al (5), Monismith et al (17), and Wang (12). Sparrow and Tory (18) found the stresses within a clay mass to be linearly proportional to the applied surface pressure, while

the strains in the mass increased more rapidly than the applied pressure, thus also indicating a stress-softening response similar to that in laboratory tests on clay. Stiffening-type load-deflection relations have been observed in tests at the surface of a structure consisting of a gravel layer over a clay subgrade (Fig. 3, Seed et al (5)). It appeared that the stiffening behavior of the gravel (consistent with that observed in laboratory tests) overrode the softening behavior of the clay, resulting in a stiffening response of the pavement as a whole. Monismith et al (17) reported a softening load-deflection response for a pavement comprising a thick asphalt concrete layer over clay. The vertical strains within the asphalt concrete were small, and the softening behavior of the subgrade was reflected at the surface. The entire range of non-linear responses was observed by Shifley (19) on a single pavement. He conducted plate tests on successive layers of a pavement during construction, observing the overall load-deflection relation to be of a marked softening type on the exposed subgrade clay; of a marked stiffening type following the addition of an 11-in. crusher-run base; almost linear when 2.4 in. of asphalt concrete were laid; and markedly softening again when the pavement was completed with another 4.8 in. of asphalt concrete. Wang (12) found that the load-deflection response of a structure with a 6 percent cement-treated silty clay over a highly plastic clay was almost linear, but the radial strains at the underside of the treated layer increased more rapidly than the plate pressure, indicating a softening response. The vertical stress at the top of the subgrade was almost linearly related to the plate pressure.

Non-linear pavement response has also been observed in measurements beneath pneumatic tires of trucks, and two examples will be given. The California Division of Highways (20) has reported extensive deflection tests on pavements with unbound aggregate, gravel, or sand bases or subbases. The majority showed a stiffening-type axle load-deflection relation, and a few, particularly those with portland cement constrains in the mass increased more rapidly than the applied pressure, thus also indicating a stress-softening response similar to that in laboratory tests on clay.

Stiffening-type load-deflection relations have been observed in tests at the surface of pavements consisting of a gravel layer over a clay subgrade (Fig. 7, Seed et al (9)). It was found that the stiffening behavior of the gravel (consistent with that observed in laboratory tests) overrode the softening behavior of the clay, resulting in a stiffening response of the pavement as a whole. Monismith et al (17) reported a softening load-deflection response for a pavement comprising a thick asphalt-concrete layer over clay; the vertical strains within the asphalt concrete were small, and the softening behavior of the subgrade was reflected at the surface. The entire range of non-linear response was observed by Shifley (19) on a single pavement. He conducted plate tests on successive layers of a pavement during construction, observing the overall load-deflection relation to be of a marked softening type on the exposed subgrade clay; of a marked stiffening type following the addition of an 11-in. crusher-run base; almost linear after 2.5 in. of asphalt concrete were laid; and markedly softening again when the pavement was completed with another 4.5 in. of asphalt concrete. Wang (12) found the load-deflection response of a structure with a 6 percent cement-treated silty clay over a highly plastic clay was almost linear, but the radial strains at the underside of the treated layer increased more rapidly than the plate pressure, indicating a softening response. The vertical stress at the top of the subgrade was almost linearly related to the plate pressure.

Non-linear pavement response has also been observed in measurements of other quantities of interest, and two examples will be given. The California Division of Highways (20) has reported extensive deflection tests on pavements with untreated aggregate, gravel, or sand bases or subbases. The majority showed a stiffening-type load-deflection relation, and a few, particularly those with portland cement concrete or cement-treated layers, showed approximately linear relations. In a test track pavement comprising 12 in. of rolled asphalt over clay, Lister and Jones (21) observed the surface deflection and the vertical stress on the subgrade to be linearly related to the wheel load. In a pavement with 4 in. of rolled asphalt and an unbound aggregate base over the clay, however, the surface deflection and subgrade vertical stress increased more slowly than the wheel load, suggesting that the increasing stiffness of the aggregate base overrode any decrease in stiffness of the clay with increasing wheel load. Finally, an almost linear response was observed in a pavement with a cement-bound granular base.

The non-linearity exhibited by pavement structures thus varied from the softening to the stiffening type, depending on the materials of construction, and this behavior was generally consistent with that which would be expected considering the non-linear response of the individual materials. The non-linearity of the field load-deflection relations was however generally much less pronounced than that of the constituent materials in the laboratory.

STRESS AND STRAIN ANALYSIS FOR PAVEMENTS WITH NON-LINEAR MATERIALS

In the analysis of stresses and strains induced by traffic the assumption is frequently made that the pavement can be represented by a half-space with homogeneous isotropic linear elastic layers, and that the traffic loading can be represented by uniform normal pressures applied over one or more circular areas on the surface of the half space. Computer solutions, using integral transform techniques, have been published by Chevron (Warren et al (22)) and Shell (Peutz et al (23)). Finite element techniques developed in recent years permit more general analysis of linear elastic pavement structures (Waterhouse (24), Duncan et al (25), Westmann (26)). The adaptation of standard finite element procedures to pavements is generally straightforward, but as

the technique is an approximate one, the element configuration and boundary conditions must be carefully selected to optimize the results (16).

Attempts have been made over a long period to account for non-linearities in the response of real materials by ad hoc modifications to linear elastic solutions. The resulting solutions, however, did not satisfy one or other of the governing equations, and were recognized as being only rough approximations. The Griffith and Frohlich (27) concentration factors have been applied with varying success to modify the Boussinesq solution to fit the greater concentration of stresses and displacements (due in part to non-linear response) observed in tests on sand masses. Vesic (28) found that the subgrade stresses in a pavement with a granular base were given more accurately by the Boussinesq solution than by the solution for the layered structure (using moduli measured in the laboratory under compressive stresses) and attributed this to non-linearity, or a difference between the tensile and compressive moduli of untreated granular materials. He introduced an ad hoc modification to the Boussinesq solution to predict the deflections. Seed et al (5) and Kasianchuk (29) used an iterative procedure with integral transform solutions for layered linear elastic systems to take account approximately of variations in material moduli with depth resulting from non-linearity, but were unable to account for variations with radial offset using this technique. With finite element methods some of these approximations can be avoided, and more strictly correct solutions are possible. Solution algorithms which have been applied to boundary value problems associated with non-linear elastic materials involve obtaining a series of solutions for linear problems, and using iterative or incremental techniques (Duncan et al (25), Hudson et al (30)).

Analysis of Pavements with Granular Layers

The most pronounced effect of non-linear material response on the overall behavior of a pavement is likely to be found in a structure in which the upper layers are formed of unbound granular materials.

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...ents with Granular Layers

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Duncan et al (25) used an iterative finite element technique to analyze a pavement made up of a 6.6 in. asphalt concrete surfacing, a 21 in. granular base and subbase, and a clayey sand subgrade, loaded through a 12 in. diameter plate. The simplified constitutive equations used were based on the results of repeated flexure and uniaxial compression tests in the laboratory. The asphalt concrete was taken as linear, the modulus of the clayey sand obtained from a curve showing its variation with principal stress difference, and the moduli of the granular bases taken as a function of the minor principal stress (see Equation 2). In order to avoid problems as the modulus of the granular material tended to zero at low or tensile minor principal stresses, a lower limit of the modulus was assumed. Two analyses were carried out, one corresponding to winter conditions when the stiffness of the asphalt concrete was high, and one corresponding to summer conditions, with low stiffnesses. For winter conditions, the stresses in the granular base were compressive throughout. For summer conditions however, a tendency for tension to develop in the granular base resulted in very low moduli in the zone directly beneath the tire. Duncan (31) subsequently reanalyzed this pavement using an incremental technique, and this provided more stable solutions, indicating that the stresses in the granular base were compressive even during summer, although the moduli in this layer were still lower than those during winter. The surface deflections for the summer condition increased more sharply near the loaded area than did those for the winter condition. In the summer condition the base compression contributed more to the total deflection than it did during the winter. The incremental analysis revealed the relation between the applied load and the surface deflection to be practically linear.

In order to explore further the effect of non-linear response of granular materials, a hypothetical uniform sand mass subjected to a normal pressure uniformly distributed over a circular area on its surface was analyzed using an axisymmetric finite element

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... the effect of non-linear response of granular materials ... mass subjected to a normal pressure uniformly distributed ... on the surface was analyzed using an axisymmetric finite element procedure (16). This probably represents the extreme degree of non-linearity likely to be of interest to the pavement engineer.

A correctly-formulated constitutive equation for sands is not currently available, and an approximation was necessary. As described previously, several research investigations have shown that the resilient moduli of sands can be expressed by an equation of the form of (1), independent of the deviatoric stress (in the absence of shear failure). The expression

$$M_R = 10,000\,(I_1)^{0.5} \tag{4}$$

was adopted, the exponent of 0.5 being one frequently reported, and the coefficient of 10,000 psi being unimportant for present purposes. As the finite element program became inaccurate at very low moduli, a lower limit of 1,000 psi was assigned, corresponding to the modulus at a value of I_1 (sum of the normal stresses) of 0.01 psi. The laboratory studies on sands described previously were generally based on uniaxial tests, and little information consequently exists on initial and stress-induced anisotrophy, or on the Poisson coefficients and their variation with stress level. Lacking this information, the materials was assumed to behave isotropically, and to have a constant Poisson's ratio of 0.3. A realistic density of 108 lb/cu. ft was used, and the coefficient of earth pressure at rest taken as 0.43. The finite element mesh adopted is shown in Fig. 4. The surface load was applied in six equal increments, corresponding to pressure increments of 5 psi. The initial moduli in each element were computed from (4), for the gravity stress condition. Moduli for successive increments were computed from the stresses yielded by the finite element analysis after the application of the previous increment. The increases in vertical displacements at various depths in the sand mass with successive load increments, as given by the finite element analysis, are shown in Fig. 5. A distinctly non-linear stiffening-type load-deflection relation is apparent in the case of the surface deflections. The load-

deflection relations at greater depths, where the constant gravity stresses form a greater proportion of the total stresses, are more linear. The vertical stresses in the mass beneath the center of the loaded area, and their variation with the applied surface pressure, are shown in Fig. 6. The stresses near the surface are of course almost equal to the applied pressure. The stresses at greater depths are not linearly related to the surface pressure, but the non-linearity is less pronounced than in the case of the deflections (Fig. 5). The variation of vertical stress with depth is shown in Fig. 7, where it is compared with that yielded by other theoretical solutions. Many experimental investigations of sands have revealed that the vertical stresses indicated by linear elastic theory are less than those measured, and the Frohlich (27) modification to the linear solution (with a concentration factor of 4) has sometimes been reported to approximate observed stresses more accurately. As seen in Fig. 7, the non-linear solution gives stresses between those yielded by the linear and Frohlich ($n = 4$) solutions.

One of the important factors not taken into account in the analysis was the anisotropy of behavior of the sand. Several authors have reported an initial cross-isotropy in sands, with the stiffness in the vertical direction exceeding that in the horizontal directions. Analyses by Barden (32), Gerrard and Mulholland (33) and others have indicated that the effect of such anisotropy is to increase the vertical stresses in a mass near the axis of loading. Borowicka (34), considering the increase in the modulus of a sand mass with depth resulting from the increase in gravity stresses, also obtained an increase in the stresses on the axis of loading. It seems likely that these properties of sand, namely the initial cross-isotropy and initial variation of stiffness with depth (functions of gravity stresses, the coefficient of earth pressure at rest, and residual stresses), as well as any load-induced anisotropy and variations in stiffness from point to point in the mass, stem from the single effect of non-linear response of the sand. Instead of using separate analyses to take account of anisotropy [illegible] at greater depths, where the constant gravity stresses form a [illegible] proportion of the total stresses, are more linear. The vertical stresses in [illegible] beneath the center of the loaded area, and their variation with the applied [illegible] magnitude, are shown in Fig. 6. The stresses near the surface are of course [illegible] applied pressure. The stresses at greater depths are not linearly [illegible] the surface pressure, but the non-linearity is less pronounced than in the [illegible] (Fig. 5). The variation of vertical stress with depth is shown [illegible] compared with that yielded by other theoretical solutions. Many [illegible] of sands have revealed that the vertical stresses indicated [illegible] linear theory are less than those measured, and the Frohlich (27) modifica- [illegible] with a concentration factor of 4) has sometimes been re- [illegible] stresses more correctly. As seen in Fig. 7, the [illegible] between those yielded by the linear and Frohlich [illegible]

[illegible] not taken into account in the analysis was the anisot- [illegible] of the sand. Several authors have reported an initial cross-isotropy [illegible] in the vertical direction exceeding that in the horizontal [illegible]. Analyses by [illegible] (22), Gerrard and Mulholland (23) and others have [illegible] that the [illegible] of such anisotropy is to increase the vertical stresses in a [illegible] loading. Borowicka (24), considering the increase in the modulus [illegible] depth resulting from the increase in gravity stresses, also ob- [illegible] the stresses on the axis of loading. It seems likely that these [illegible] namely the initial cross-isotropy and initial variation of stiffness [illegible] of gravity stresses, the coefficient of earth pressure at rest, [illegible] stresses), as well as any load-induced anisotropy and variations in stiff- [illegible] the mass, stem from the single effect of non-linear re- [illegible]. Instead of using separate analyses to take account of anisotropy and stiffness variations as in the past, it would seem feasible to account for all of these using a single non-linear elastic finite element analysis with a correctly formulated constitutive equations, but as mentioned previously, such an equation is not available at present.

In addition to the errors due to neglect of the anisotropy of response, the above analysis must contain errors of unknown magnitude resulting from local particle slip which must tend to occur in elements where the rupture criterion for the sand is approached. Finally, errors are present due to the finite sizes of the elements, and the limited number of increments in which the load was applied. The analysis does however give an indication of the likely effects of non-linear material response in a pavement structure comprised essentially of granular materials, such as might be used for highways carrying light or medium traffic.

Analysis of Full-Depth Asphalt Concrete Pavements

The full-depth asphalt concrete pavement is gaining popularity for heavily-trafficked highways, and one such pavement will be studied in this section.

The full-scale road experiment on Sweetwater Road in San Diego is a cooperative project involving the Asphalt Institute, the County of San Diego, Materials Research and Development Inc., the State of California, and the University of California (Kingham (35)). Test Section 2 is one of the full-depth asphalt concrete sections, having a 3 in. asphalt concrete surfacing, and a 5.8 in. asphalt concrete base, overlying about 27 ft of variable brown sandy clay (Fig. 8). During April 1968 Materials Research and Development Inc., with the aid of the County of San Diego, carried out a series of tests on this section as part of a comprehensive program of periodic investigations. The tests included measurements of transverse and longitudinal strains (at the surface and at two depths within the asphalt concrete), surface deflections, and the vertical compression within the asphalt concrete, induced in the outer wheel path [illegible] these [illegible] cases, it would seem [illegible] to [illegible] from a single non-linear elastic finite element analysis with [illegible] [illegible] constitutive equations, but as mentioned [illegible] such [illegible] are not available at present.

In addition to the errors due to neglect of the anisotropy of response, the [illegible] analysis must contain errors of unknown magnitude resulting from local particle slip which must tend to occur in elements where the rupture criterion for the sand is approached. Finally, errors are present due to the finite sizes of the elements, and the limited number of increments in which the load was applied. The analysis does however give an indication of the likely effects of non-linear material response in a pavement structure comprised essentially of granular materials, such as might be used for highways carrying light or medium traffic.

Analysis of Full-Depth Asphalt Concrete Pavements

The full-depth [illegible] [illegible] pavement [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] and [illegible] pavement will be studied in this section.

The [illegible] [illegible] experiment on [illegible] Road in San Diego [illegible] [illegible] [illegible] by the Asphalt Institute, the County of San Diego, Materials [illegible] Development [illegible] the State of California, and the University of California.

[illegible] (25). Test Section 2 is one of the full-depth asphalt concrete sections, [illegible] a [illegible] asphalt concrete surfacing, and a [illegible] in. asphalt concrete base, over [illegible] [illegible] of variable brown sandy clay (Fig. 9). During April 1968 Materials [illegible] and Development [illegible], with the aid of the County of San Diego, carried out [illegible] of tests on this section as part of a comprehensive program of periodic [illegible]. The tests included measurements of transverse and longitudinal strains [illegible] the surface and at two depths within the asphalt concrete), surface deflections, and [illegible] compression within the asphalt concrete, induced in the outer wheel path by a standard truck moving at 5 mph. Undisturbed 4 in. diameter samples were recovered from the top 6 ft of the subgrade, and 4 in. diameter cores of the asphalt concrete taken from an adjacent full-depth section. Detailed material characterization tests were carried out on these samples in the laboratory, and the results have been summarized briefly in a previous section.

Analyses were made of the stresses, strains, and displacements induced in the asphalt concrete and subgrade by a single front wheel of the test truck. The wheel load was approximated by a uniform normal pressure over a circle of the same area as the tire contact area (pressure 51.5 psi, radius 3.4 in.). All layers were assumed to be of large horizontal extent, and this permitted the use of axisymmetric analytical procedures. For purposes of comparison, the structure was analyzed assuming the materials to behave both linearly (Shell and finite element solutions) and non-linearly (finite element solution).

In the case of the Shell Analysis, the asphalt concrete was subdivided into five layers with different stiffnesses to represent approximately the continuous variation of stiffness with depth corresponding to the temperature profile measured in-situ with thermocouples. The subgrade was divided into five layers, corresponding to the depths of the undisturbed samples. The assumption was made that the subgrade extended semi-infinitely below the lowest sample (at a depth of 6.5 ft), with the same properties as those of this sample. Sampling to greater depth would have been desirable. Linear isotropic elastic coefficients were required for each layer, and values were selected which might be reasonably representative of the spectrum of coefficients in an actually non-linear material, and which could also be measured in simple laboratory tests. In the case of the subgrade, these were the resilient modulus and resilient Poisson's ratio (both secant values) measured in the triaxial apparatus under an all-round sustained pressure corresponding to the vertical gravity pressure at the sampling depth, and a superimposed repeated axial compressive stress of 3 psi. (The resilient moduli [illegible] undisturbed 4 in. diameter samples were [illegible] the subgrade, and 4 in. diameter cores of the asphalt [illegible] full-depth sections. Detailed material characterization [illegible] these samples in the laboratory, and the results have [illegible] previous section.

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[illegible] analysis, the asphalt concrete was subdivided into five layers [illegible] the continuous variation of stiffness [illegible] to the temperature profile measured in-situ with thermo- [illegible] into five layers, corresponding to the depths of the [illegible] assumption was made that the subgrade extended semi- [illegible] (at a depth of [illegible] ft), with the same properties as [illegible] greater depth would have been desirable. [illegible] required for each layer, and values were selected [illegible] of the spectrum of coefficients in an actual [illegible] which could also be measured in simple laboratory tests. [illegible] were the resilient modulus and resilient Poisson's [illegible] measured in the triaxial apparatus under an all-round [illegible] the vertical gravity pressure at the sampling depth, [illegible] axial compressive stress of 5 psi. (The resilient moduli

...wn in Fig. 8). The asphalt concrete moduli were those for zero sustained ... and a repeated axial compressive stress of 20 psi. The asphalt concrete had tested in the laboratory at only two temperatures (70° and 94°F), and it was thus ...essary to estimate the stress-strain coefficients for the range of temperatures ...easured during the field tests. This was done using a linear interpolation on a plot ...f temperature against the logarithms of the stiffness coefficients. The linear finite element analysis was carried out using the same material properties and layer thicknesses. The finite element mesh employed is shown in Fig. 9.

The selection of material properties for use in the non-linear finite element analysis was more difficult. While the triaxial tests conducted had adequately defined the constitutive equations for the materials at points beneath the center of the axisymmetrically loaded area, the results were inadequate for conditions elsewhere in the structure (because the triaxial test permits only two normal stresses to be varied independently, and the resulting two strains to be measured, while for complete characterization for use with an axisymmetric boundary value problem three normal and one shear stress would have to be varied independently, and the resulting four strains measured). In order to use the triaxial test results in making an approximate analysis of a pavement it is therefore necessary to generalize them. Unfortunately, engineering judgment has to be invoked to do this, as it has been in all previous research studies involving material tests and pavement analysis. The initial and stress-induced anisotropy exhibited by both the clay and asphalt concrete could not be separated accurately, and it was thought best to make the simplifying assumption of isotropic behavior. Reasoned assumptions were also required to relate the laboratory reference stress states to the field reference stress states. These have been described elsewhere (16). In the computer analysis, where the reference stress state in an element was different from that ...hich the laboratory tests had been performed, the program interpolated the two ...aterial coefficients using polynomial regression. The finite element

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...the stress-strain coefficients for the range of temperatures
...at the two temperatures (70 and 95°F), and it was thus
...compressive stress of 20 psi. The asphalt concrete had
...concrete moduli were those for zero surcharge.

mesh used was similar to that in Fig. 9. The surface tire pressure was applied in five equal increments, and the computational procedure was similar to that described previously for the sand mass.

Fig. 10 shows the variation of vertical deflections beneath the center of the loaded area with increasing applied pressure, as indicated by the non-linear finite element analysis. In Test Section 2, where the clay subgrade was relatively stiff, the deflection at the top of the subgrade is almost linearly related to the tire contact pressure. In the case of another full-depth asphalt concrete pavement, Test Section 35, where the upper portion of the subgrade was a highly resilient silty clay, there is a softening-type relation between the contact pressure and subgrade deflection, typical of that reported in previous experimental studies on pavements with clay subgrade (5, 12, 17, 19). In both cases the load-deflection relation at the surface is almost linear, or slightly of the stiffening type. Fig. 11 shows the pattern of variation of surface deflection with the distance from the center of the load, and Fig. 12 the corresponding pattern for the compression within the asphalt concrete layers (i.e. the difference between the surface and top-of-subgrade deflections). The surface deflections indicated by the non-linear analysis are more concentrated about the axis of loading than those given by the linear analyses.[5] Comparing the two finite element solutions, the non-linear analysis yields a maximum deflection 13 percent larger and a compression within the asphalt concrete layers 25 percent larger, than those given by the linear analysis. Finally, the maximum surface deflections indicated by the theoretical analyses were 17 percent to 32 percent larger than that measured. This agreement is considered fair. Less satisfactory, however, is the marked difference in the deflection patterns,

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[5] The surface deflections yielded by the linear finite element analysis are consistently less than those given by the linear Shell analysis. This was due to the finite depth to the lower boundary (15 ft) which had to be used in the finite element analysis, rather than the infinite depth used in the Shell half-space solution.

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[illegible] shows the variation of vertical deflections beneath the center of the loaded [illegible] with increasing applied pressure, as indicated by the non-linear finite element [illegible]. In Test Section 2, where the clay subgrade was relatively stiff, the deflec- [illegible] at the top of the subgrade is almost linearly related to the tire contact pressure. [illegible] the case of such a full-depth asphalt concrete pavement, Test Section 25, where [illegible] upper portion of the subgrade was a highly resilient silty clay, there is a soften- ing-type relation between the contact pressure and subgrade deflection, typical of that [illegible] in previous experimental studies on pavements with clay subgrade (5, 12). [illegible] In both cases the load-deflection relation at the surface is almost linear, [illegible] of the stiffening type. Fig. 11 shows the pattern of variation of surface de- [illegible] with the distance from the center of the load, and Fig. 12 the corresponding [illegible] for the compression within the asphalt concrete layers (i.e. the difference be- tween the surface and top-of-subgrade deflections). The surface deflections indicated [illegible] non-linear analysis are more concentrated about the axis of loading than those [illegible] linear analyses.* Comparing the two finite element solutions, the non- [illegible] yields a maximum deflection 15 percent larger and a compression with- [illegible] concrete layers 25 percent larger, than those given by the linear analysis. [illegible] the maximum surface deflections indicated by the theoretical analyses were [illegible] 32 percent larger than that measured. This agreement is considered [illegible] less satisfactory, however, in the marked difference in the deflection patterns.

* [illegible] deflections yielded by the linear finite element analysis are consistently [illegible] than those given by the linear Shell analysis. This was due to the finite depth to [illegible] boundary (15 ft) which had to be used in the finite element analysis, rather [illegible] infinite depth used in the Shell half-space solution.

the theoretical deflections yielded by all three analyses being far more concentrated than those measured. No conclusive explanation for this is apparent.

Fig. 13 shows the variation with depth of the horizontal strains in the asphalt concrete at points beneath the center of the load. These patterns seem strange at first sight, but appear to result from the marked temperature gradients in the asphalt concrete at the time the field tests were conducted. There is good agreement between the two linear elastic analyses, and the strains they yield are little different from those given by the non-linear elastic analysis. The few experimental results available are included in the figure, and follow the same general pattern as the analytical results. The large difference between the transverse and longitudinal strains measured at the surface remains unexplained.

The vertical stresses in the subgrade beneath the center of the load are shown in Fig. 14. The non-linear analysis indicates higher stresses than the linear analyses, and this agrees with the greater concentration of deflections indicated by the non-linear analysis (Fig. 11). The difference in the stresses between the linear and non-linear analyses near the top of the subgrade is about 15 percent.

The relatively small differences between the results of the linear and non-linear analyses lead to the conclusion that non-linear material response is not a disqualification for the use of linear elastic theory for the practical design and evaluation of a full-depth asphalt concrete pavement over a sandy clay subgrade.[6]

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[6] Certain qualifications to this conclusion must be emphasized. It applies only to the particular materials and pavement structure studied, and will not necessarily be true for other conditions. The "linear" analyses described were based on linear stress-strain coefficient selected, using judgment, from the series of coefficients corresponding to the range of stress levels at which the non-linear materials were tested. The results of the linear analyses may have been considerably different if the material constants had been determined at arbitrary stress levels. the "non-linear" analysis was only approximate, as the laboratory tests conducted had not enabled the materials to be characterized completely. Conclusions as to the effect of non-linear material response on the pavement behavior can thus only be tentative.

A secondary aim of the study was to obtain an indication of how well a mechanistic approach (pavement analysis based on measured material properties) is able to predict the observed behavior of pavements. While the predicted and measured responses were similar, the quantitative agreement was in general not good. This was not unexpected, as there were many factors which were not taken into account adequately. The fact that the linear and non-linear analyses of the test road sections agreed fairly well, while the predicted and measured responses did not, indicates that (for the pavement studied) less error would be introduced in pavement design and evaluation by non-linearity of material response than by other assumptions made.

SUMMARY AND DISCUSSION

Laboratory studies have shown that under conditions representative of moving traffic on an in-service pavement many common highway materials (including sands, clays, asphalt-, lime-, and weakly cement-treated soils) exhibit a non-linear response to stress. The degree of non-linearity appears to range from very small in the case of portland cement concretes, to pronounced in the case of sands and unbound aggregates. Studies of pavement structures in the field have revealed more subdued non-linearities in the relations between applied load and the resulting deflections, strains, and stresses. The forms of these non-linearities have generally been as would be expected from a knowledge of the non-linearities in the behavior of the constituent materials. Additional indications of the likely effects of non-linear material response on the behavior of pavements were yielded by theoretical non-linear analyses. In the case of a uniform sand mass such an analysis revealed a pronouncedly stiffening relation between the pressure applied over a circular area and the surface deflection, and slightly non-linear relations between the applied pressure and the vertical stresses induced in the mass. The vertical stresses beneath the center of the load were a little higher than those for a linear elastic material. An approximate non-linear analysis of a [illegible] aim of the study was to obtain an indication of how well a mechanistic approach [illegible] analysis (based on measured material properties) is able to predict the [illegible] behavior of pavements. While the predicted and measured responses were [illegible] the quantitative agreement was in general not good. This was not unex[illegible] [illegible] there were many factors which were not taken into account adequately. The [illegible] the linear and non-linear analyses of the test road sections agreed fairly well, [illegible] the predicted and measured responses did not, indicates that (for the pavement [illegible]) less error would be introduced in pavement design and evaluation by non-[illegible] material response than by other assumptions made.

SUMMARY AND DISCUSSION

[illegible] studies have shown that under conditions representative of moving traffic [illegible] pavement many common highway materials (including sands, clays, [illegible], and lightly cement-treated soils) exhibit a non-linear response to [illegible]. The degree of non-linearity appears to range from very small in the case of [illegible] concretes, to pronounced in the case of sands and unbound aggregates. [illegible] structures in the field have revealed more subdued non-linearities [illegible] between applied load and the resulting deflections, strains, and [illegible] forms of these non-linearities have generally been as would be expect[illegible] knowledge of the non-linearities in the behavior of the constituent materials. [illegible] indications of the likely effects of non-linear material response on the be[illegible] of pavements were yielded by theoretical non-linear analyses. In the case of a [illegible] such an analysis revealed a pronouncedly stiffening relation be[illegible] applied over a circular area and the surface deflection, and slight[illegible] relations between the applied pressure and the vertical stresses induced [illegible] the [illegible] stresses beneath the center of the load were a little higher [illegible] a linear elastic material. An approximate non-linear analysis of a pavement consisting of asphalt concrete layers over a clay subgrade yielded an approximately linear relation between wheel load and surface deflection, and linear and non-linear analyses gave very similar distributions of the stresses and strains within the structure.

In relation to the procedures required for materials characterized as linear, the testing required to characterize a material as non-linear is considerably more complex, and the analysis of a pavement with non-linear materials also more complex. For practical pavement design, it would clearly be preferable to treat the materials as linear, if this does not involve too great a sacrifice of accuracy. Based on the literature reviews, tests, and analyses described in this paper, the linear procedures would appear reasonable in the case of portland cement concrete pavements, in pavements having bases strongly bound with cement overlying a cohesive subgrade, and in pavements comprising thick asphaltic concrete layers over a stiff cohesive subgrade.[7] Further confirmation of the validity of linear testing and analytical procedures for these pavements is however desirable. The linear procedures are not adequate in the case of pavements comprising thin surfacings over unbound aggregate, gravel, or sand bases, subbases, and subgrades.

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[7] It is again emphasized that such "linear" analyses were assumed to be based on stress-strain coefficients at carefully-selected realistic stress levels.

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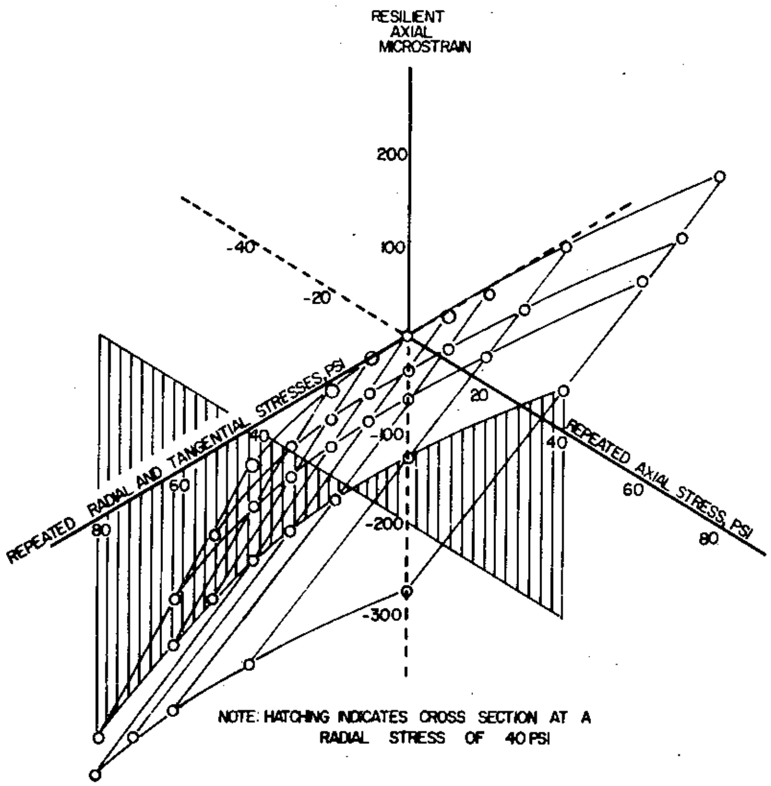


Fig. 1 – Three-dimensional surface depicting axial strains in asphalt concrete subjected to various stress states in the triaxial apparatus (70°F).

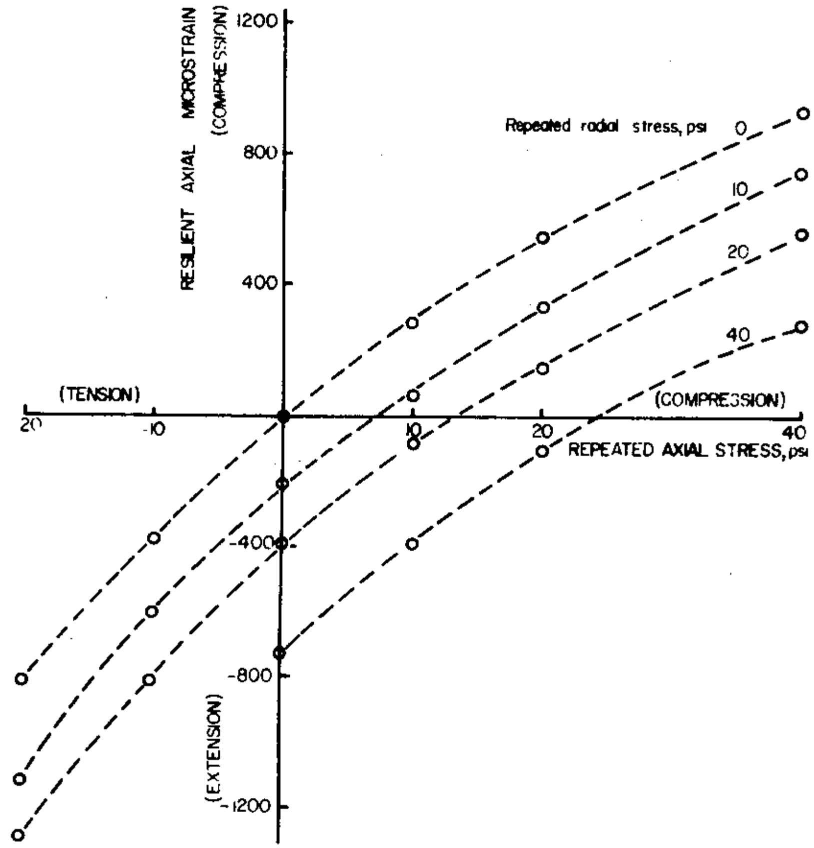


Fig. 2 – Two-dimensional representation of axial strains in asphalt concrete subjected to various stress states in the triaxial apparatus (94°F).

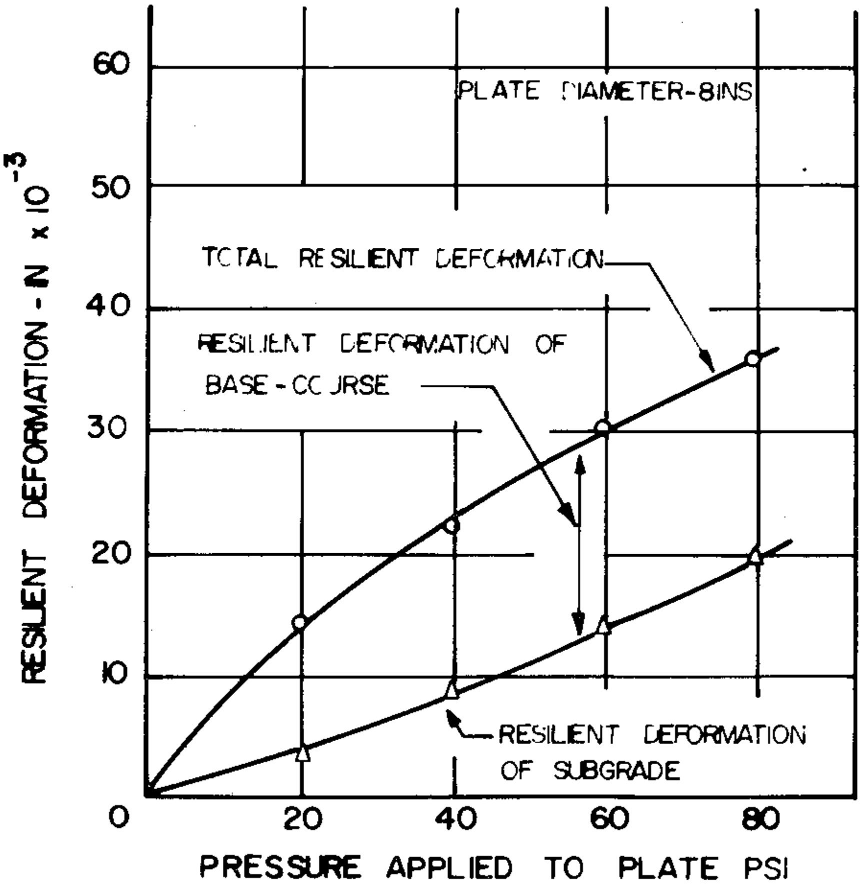
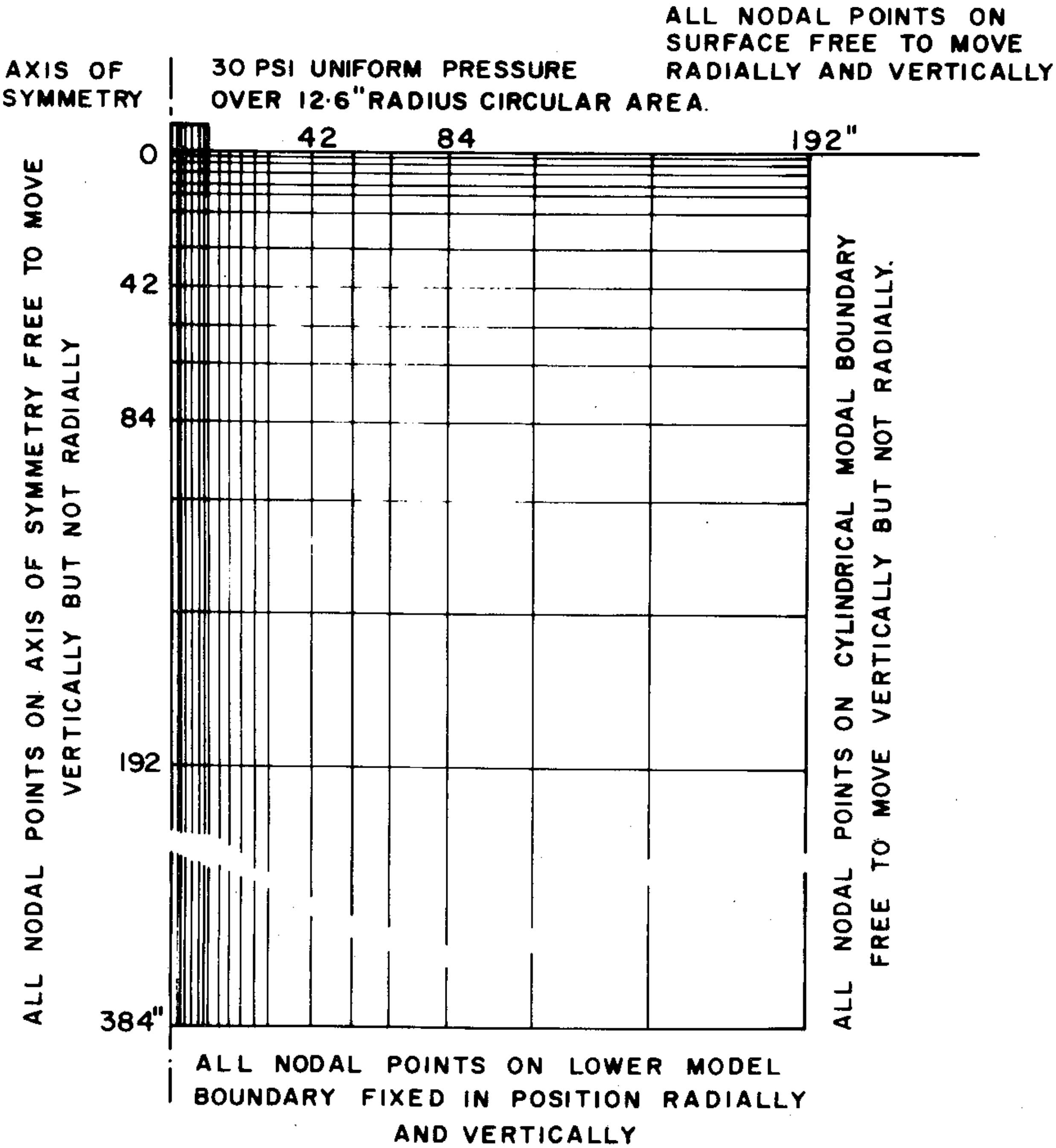


Fig. 3 — Load-deflection relationships in repetitive loading plate tests on a gravel base over a clay subgrade. (Seed et al.)



RADII TO COLUMNS OF NODAL POINTS (INCHES)
0, 1, 2, 4, 7, 10, 11·5, 12·6, 13·5, 15, 18, 22, 26, 30, 42, 54, 66, 84, 108, 144, 192

DEPTHS TO ROWS OF NODAL POINTS (INCHES)
0, 1, 2, 4·5, 7·5, 10·5, 13·5, 18, 30, 42, 54, 66, 84, 108, 144, 192, 384.

Fig. 4 – Mesh configuration and boundary conditions, finite element analysis of uniform sand.

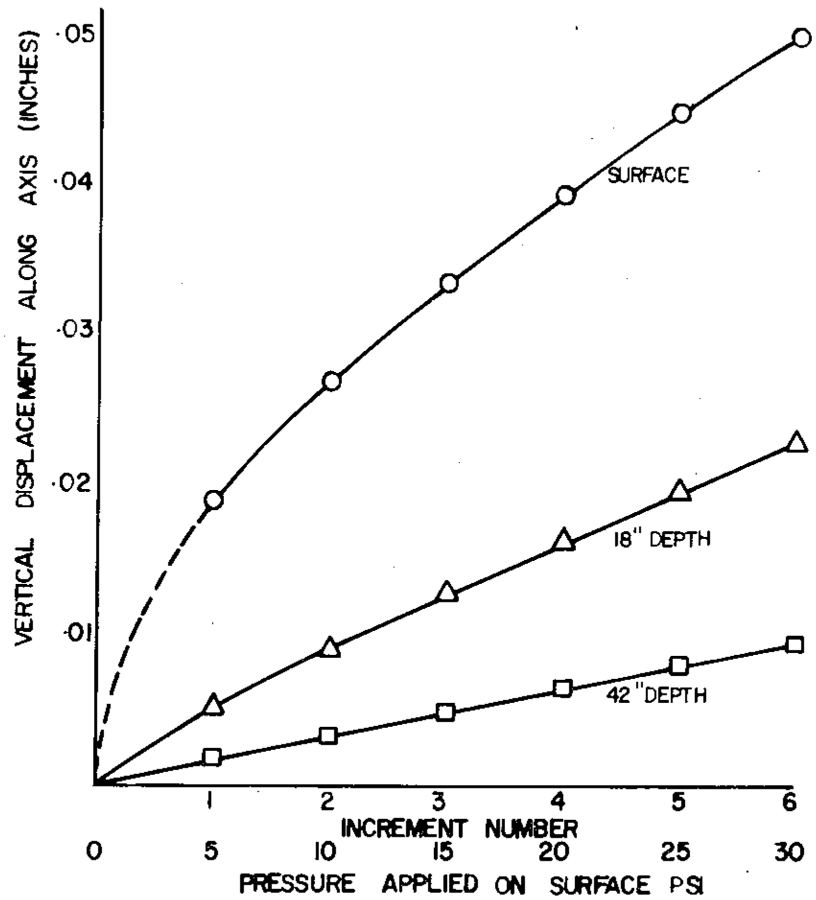


Fig. 5 — Variation of deflection with applied pressure, non-linear finite element analysis of uniform sand.

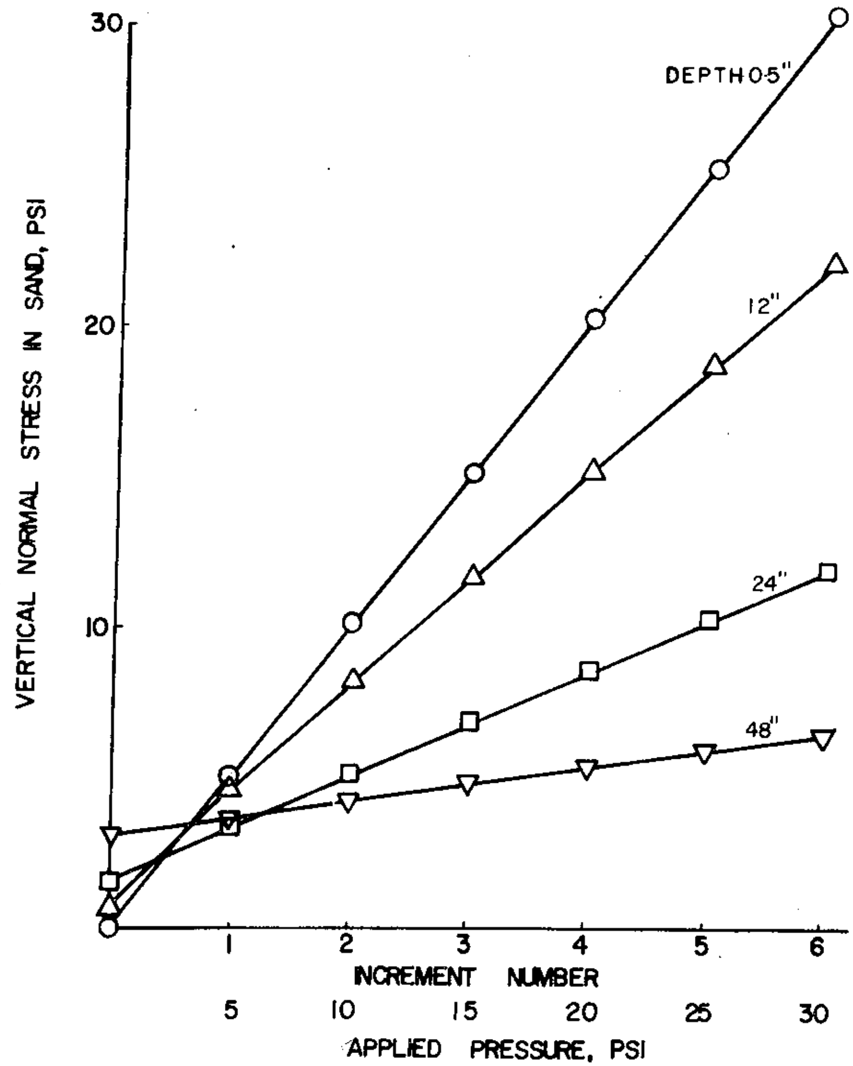


Fig. 6 – Variation of vertical normal stresses with applied pressure, non-linear finite element analysis of uniform sand. (Radial offset 0.5".)

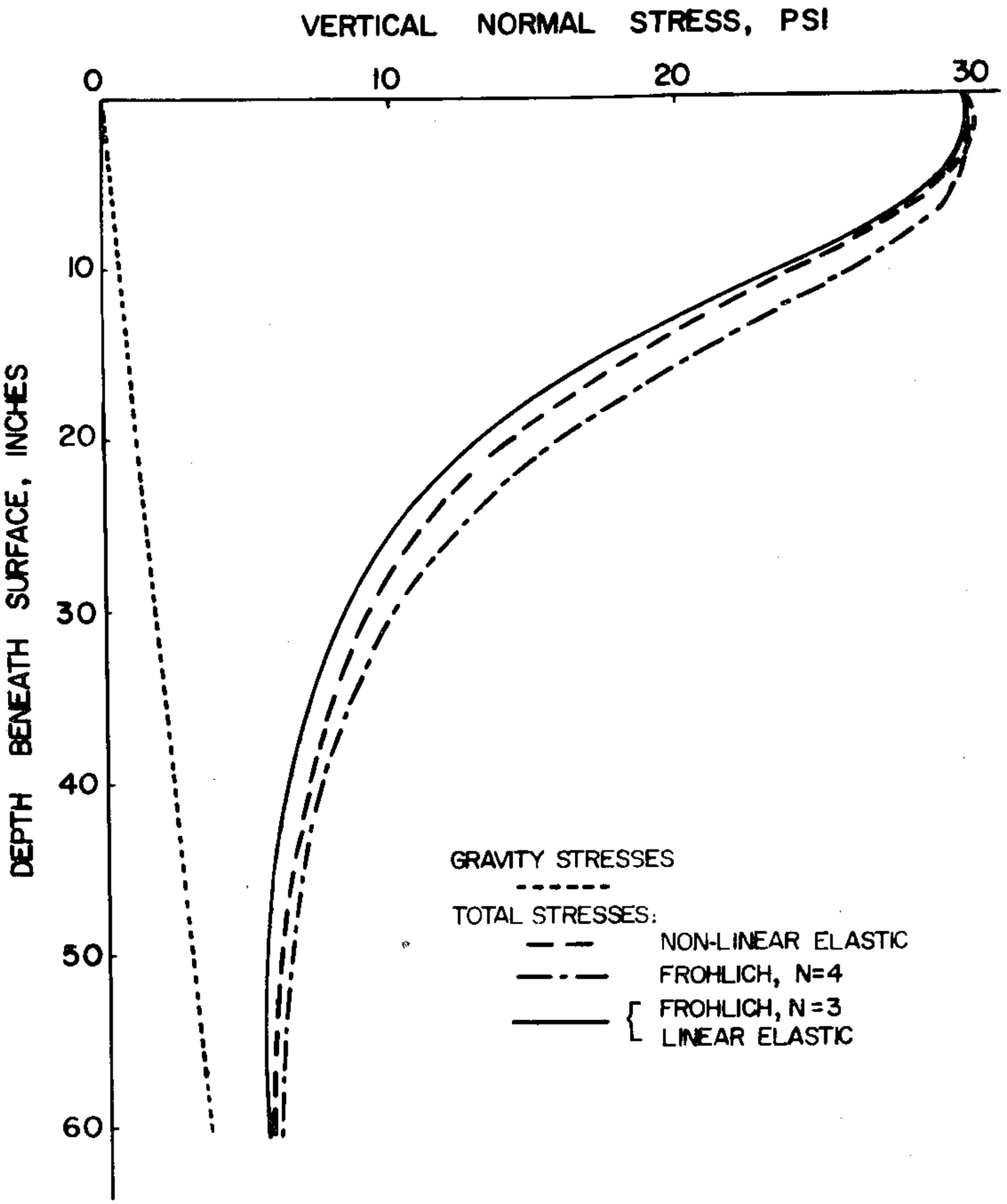
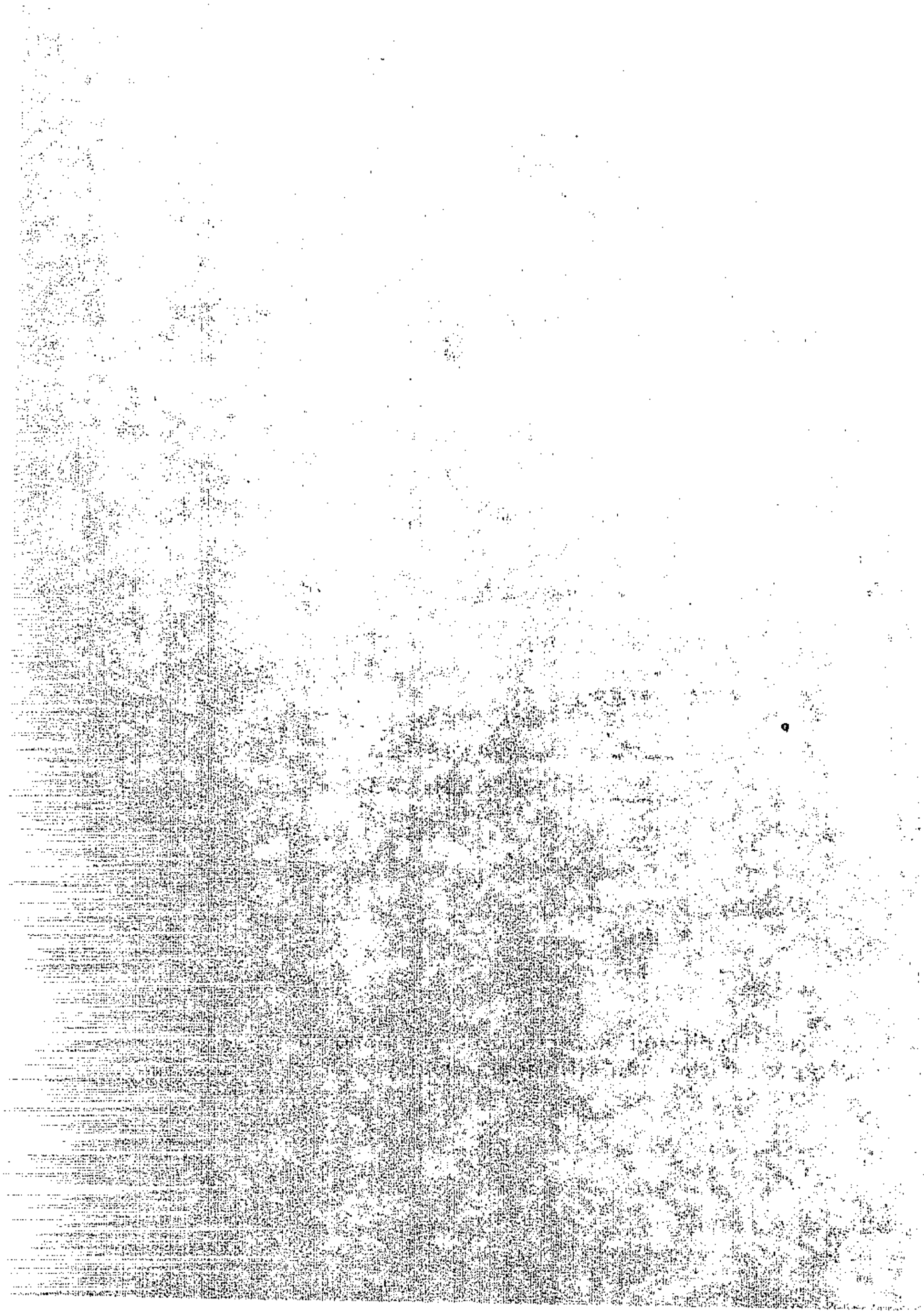


Fig. 7 – Theoretical variation of vertical stress beneath center of loaded area with depth. (Uniform sand.)



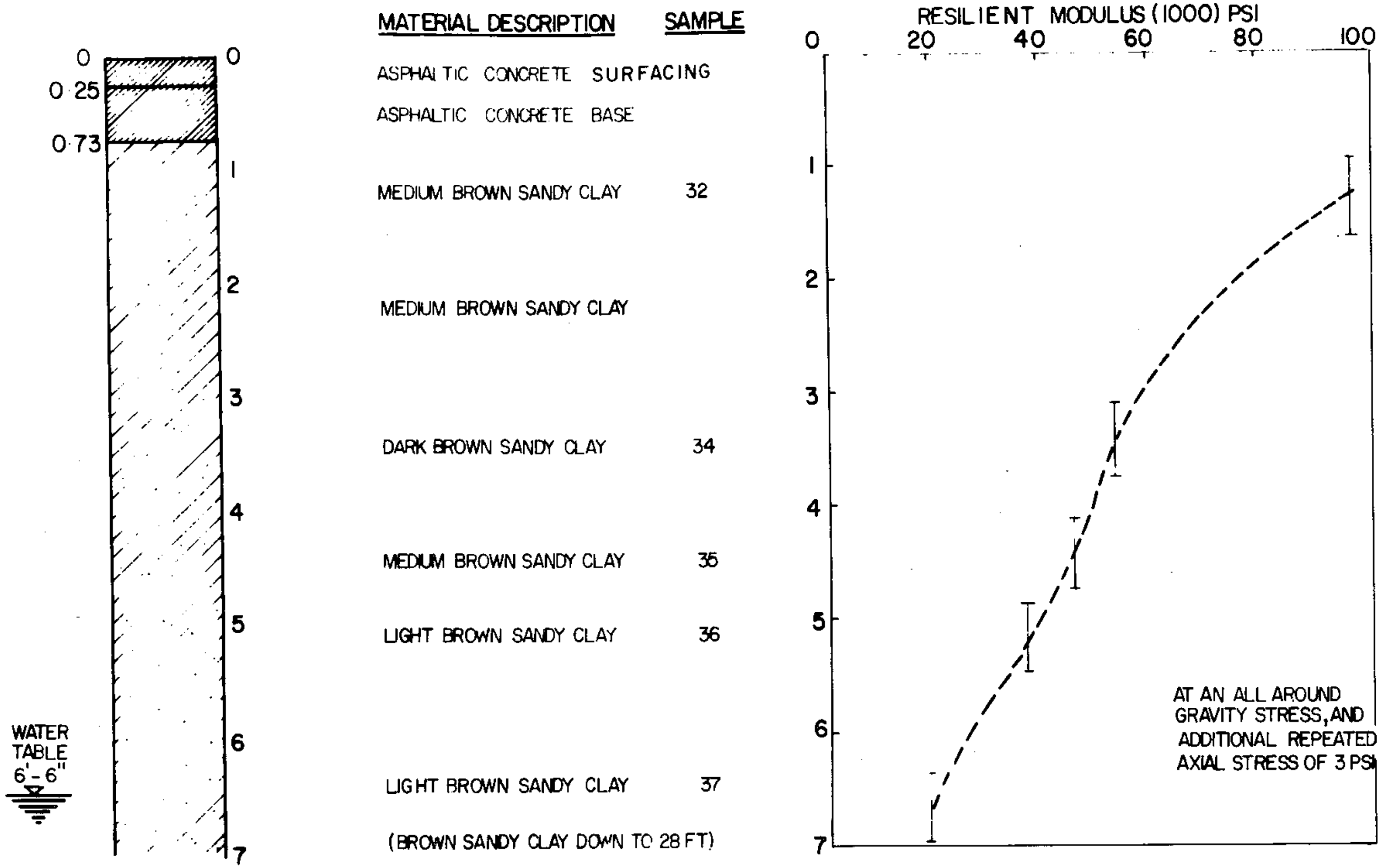
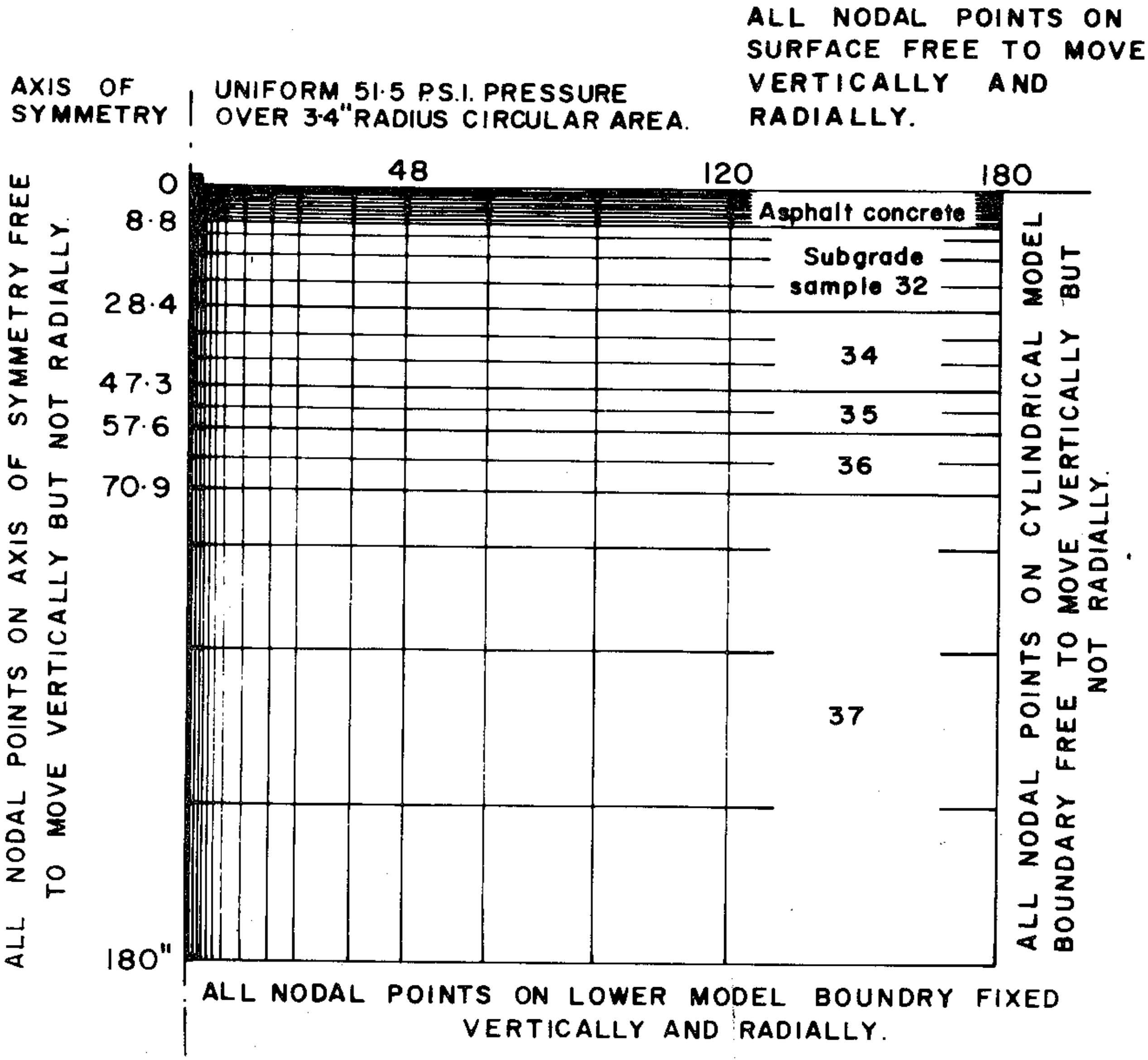


Fig. 8 – Soil profile, and representative values of resilient modulus, Section 2, San Diego Test Road.



RADII TO COLUMNS OF NODAL POINTS (INCHES)
0, 0·5, 1, 2, 3·4, 3·8, 4·8, 6, 8, 12, 24, 36, 48, 66, 90, 120, 180.
DEPTHS TO ROWS OF NODAL POINTS (INCHES)
0, 0·85, 1·7, 2·6, 3·5, 4·4, 5·3, 6·2, 7·1, 7·95, 8·8, 11·8, 16·4, 22·4, 28·4, 34·7, 41, 47·3, 52·4, 57·6, 64·2, 70·9, 84, 108, 144, 180.

Fig. 9 — Mesh configuration, boundary conditions, and material properties, linear finite element analysis, Section 2.

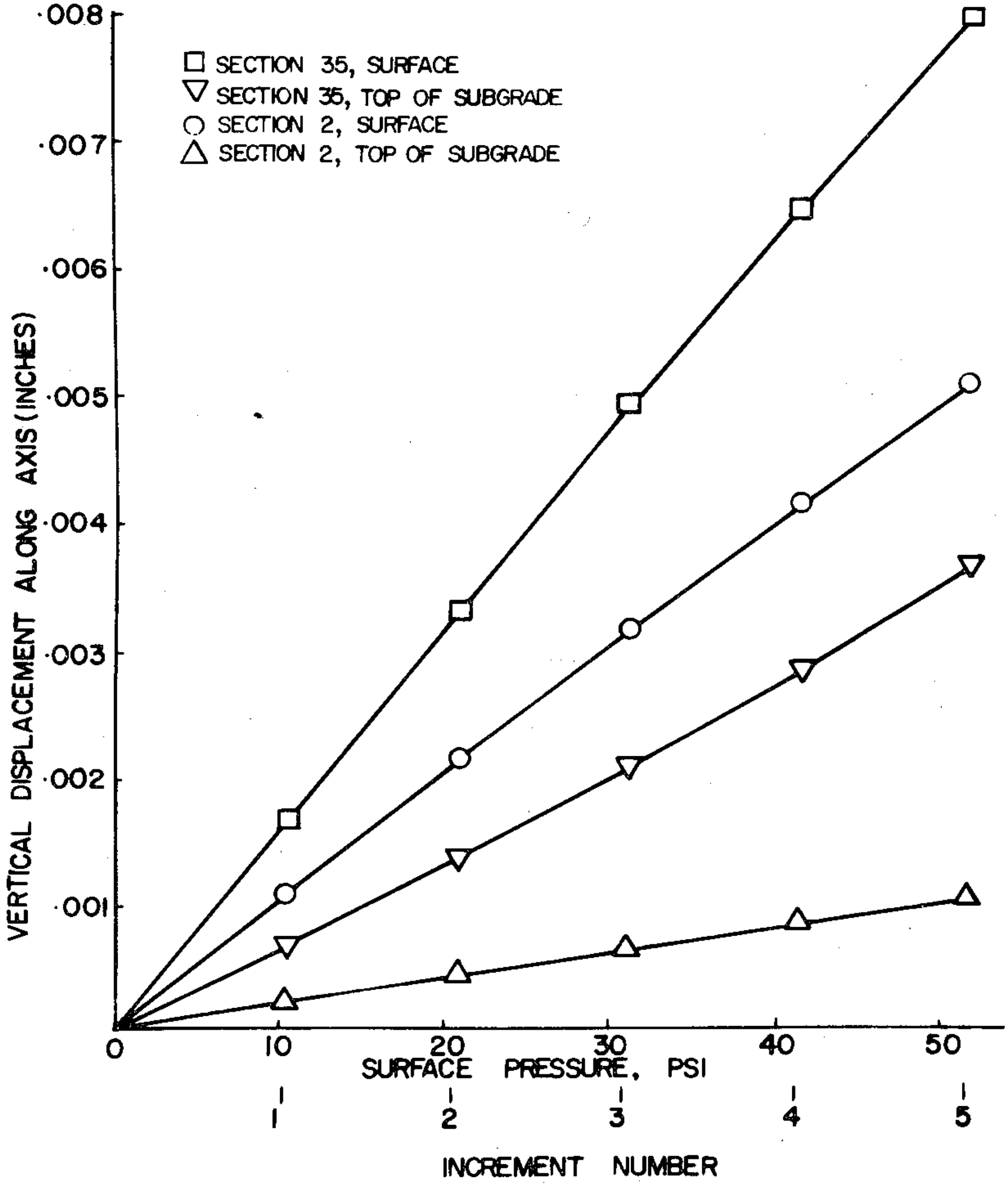


Fig. 10 – Variation of deflection with applied pressure, non-linear finite element analysis of asphalt concrete pavement.

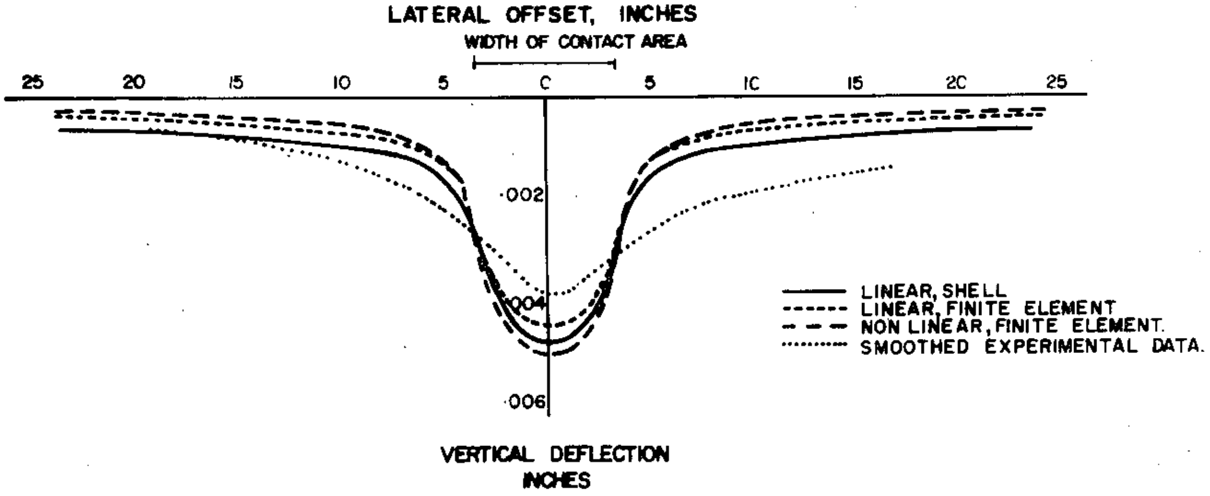


Fig. 11 – Theoretical and measured vertical surface deflections, Section 2.

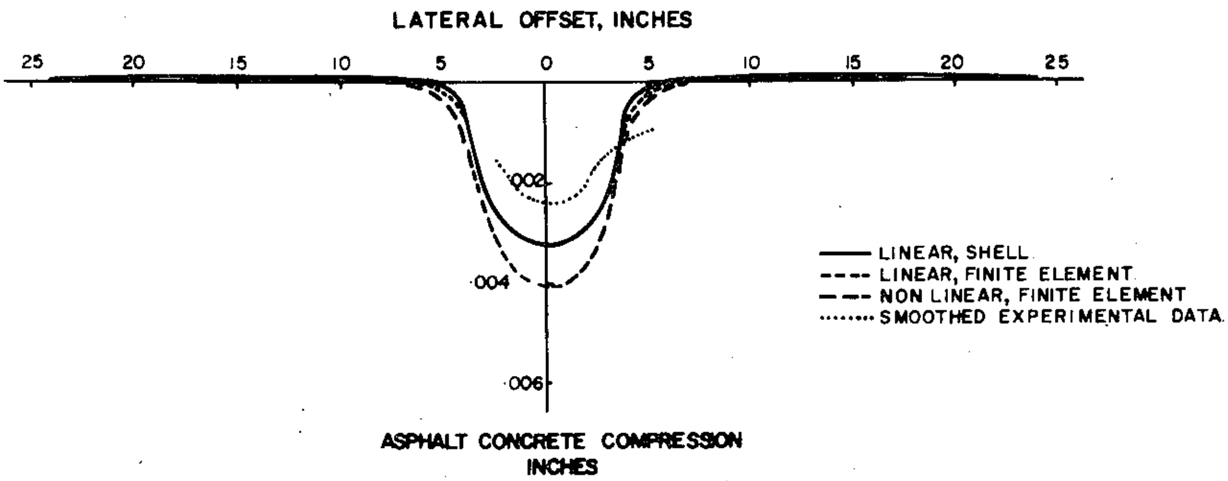


Fig. 12 – Theoretical and measured vertical compression within asphalt concrete layers, Section 2.

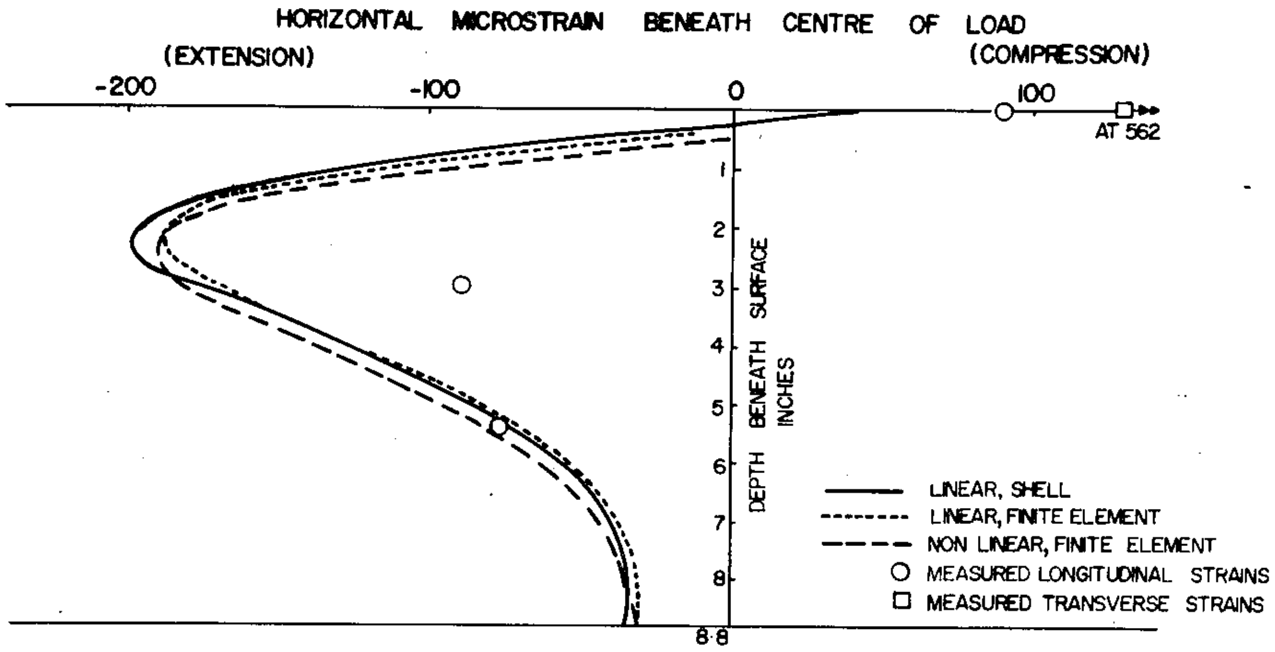


Fig. 13 – Theoretical and measured horizontal strains in the asphalt concrete, Section 2.

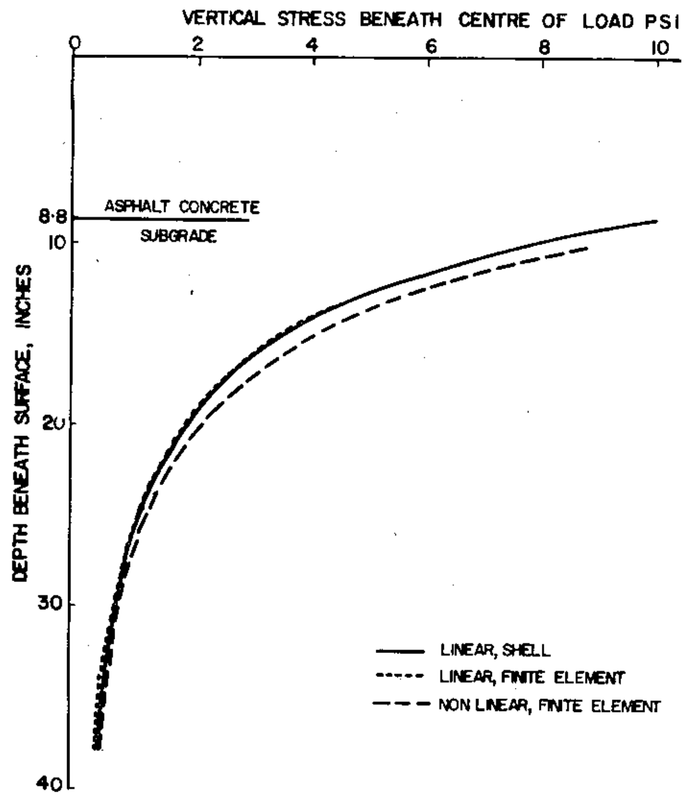


Fig. 14 – Theoretical variation of vertical stress in subgrade with depth, Section 2.