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A precision statement for the Sand Equivalent Test is computed and additional statistical treatments including the F-Ratio Test, Duncan's Multiple Range Test, and Analysis of Variance are made. A total of 4,968 Sand Equivalent Tests on three basic aggregate types (base, asphalt concrete, and Portland cement concrete) are documented. Tests were conducted by Division of Highways' personnel throughout California. The precision and uniformity of the Sand Equivalent Test, and the consumer-producer risks introduced by this test method are discussed.

The analysis involves data from five of eleven California Highways Districts and the Transportation Laboratory. Twenty-eight operators participated in the test program. Three samples from each of three material types were provided. Operators tested each sample at least twelve times. Both the Idaho and Mechanical Shakers were used in the testing program. Three replicate tests were made in a single day. Room temperature, oven dry weight, top of clay reading and top of sand reading were recorded for each replicate.

Precision statements conforming to ASTM Designation C670-71T are written for each of the three types of aggregate materials tested. Normality of data is illustrated by histograms, and test results are summarized by scatter diagrams. Further research with regard to the sources of testing bias is recommended. Precision variability due to shaker method was found to be insignificant.

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Sand equivalent test, precision, reproducibility, fine aggregates, soil testing, statistical quality control, analysis of variance

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HIGHWAY RESEARCH REPORT

PRECISION OF THE SAND EQUIVALENT TEST

INTERIM REPORT

7406

STATE OF CALIFORNIA
BUSINESS AND TRANSPORTATION AGENCY
DEPARTMENT OF TRANSPORTATION
DIVISION OF HIGHWAYS

TRANSPORTATION LABORATORY
RESEARCH REPORT
CA-DOT-TL-1153-1-74-06

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DEPARTMENT OF TRANSPORTATION
DIVISION OF HIGHWAYS
TRANSPORTATION LABORATORY
5900 FOLSOM BLVD., SACRAMENTO 95819

February 1974
FHWA Item No. F-4-20
Trans. Lab. No. 631153-1

Mr. R. J. Datel
State Highway Engineer

Dear Sir:

Submitted for your consideration is an interim report on the research project "Precision of Test Methods," entitled:

PRECISION OF

THE SAND EQUIVALENT TEST

By

Ralph R. Svetich, P.E. and Paul E. Benson, Co-Authors

R. L. Donner, P.E. and S. N. Bailey, P.E., Co-Investigators

W. H. Ames, P.E., Supervisor

Very truly yours,

JOHN L. BEATON
Chief, Transportation Laboratory

Attachment

ACKNOWLEDGEMENTS

The authors wish to express their appreciation to the Transportation Laboratory personnel and all Transportation District Laboratory personnel who conducted the numerous Sand Equivalent Tests used as the basis for this report.

Special thanks go to Mr. John Vail, who coordinated district participation for this study, the late Professor Harry Beneson, who served as Consultant, and Mr. Charles Frazier, who wrote the computer programs required to analyze the large body of data collected. We, also, extend our appreciation to Mr. George B. Sherman and Mr. Robert E. Smith of the Pavement Section whose contributions were valuable in the preparation of this report.

This project was conducted in cooperation with the Federal Highway Administration under Agreement No. F-4-20. The contents of this report reflect the views of the Transportation Laboratory which is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the State of California or the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

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INTRODUCTION

A precision statement is a measure, in the prescribed units, of the maximum difference expected between two random test results selected from the same test portion. Inherent imperfections in sampling, testing equipment and procedures are contributing factors to the magnitude of this difference. It is independent, however, from variations in materials production. Usually, precision is expressed as both a single operator's ability to repeat his results and a number of laboratories' ability to reproduce their results.[1]

If supported by a properly conducted statistical experiment designed to minimize materials variations, the actual test variability or precision encountered in the laboratory and field can be closely approximated. The variability is not specifically defined for all of the test methods currently being used in California. With contractors and producers doing more of their own testing, it is apparent that Precision Statements are needed for most contract control tests. In fact, testing procedures have been questioned in the past, and in some cases have led to claims being made against the Division of Highways. By determining quantitatively and stating clearly the precision of test methods, fewer such claims should arise and disputes over testing should be resolved more easily. In addition, designers, construction engineers, and contractors will benefit from the knowledge of testing variation contained in a Precision Statement by obtaining a better understanding of the testing risks involved in the acceptance and rejection of materials.

The Sand Equivalent Test is intended to serve as a rapid field measurement of the relative amounts of detrimental fine dust or claylike materials in soils and fine aggregates. It is the first test to be studied under this Precision Statement Project. Consequently, a thorough analysis of the data is made so that future Precision Statement experiments can benefit from the statistical techniques learned and applied in this study.

The basic aim of this study phase is to arrive at an accurate measure of test precision for the Sand Equivalent Test. A large number of identical samples were needed to accomplish this. Tests were run on these samples by different operators, shaker methods and districts. These results were categorized and analyzed by a statistical technique known as Analysis of Variance (see Appendix B). This technique provides a way of

[1]The general concepts of repeatability and reproducibility are explained in the Glossary (Appendix A).

comparing results obtained under different treatments or effects (i.e. operators, shaker method). The analysis will indicate which effects produce significant differences in results. It also provides a quantitative measure of the variation in results. This measure was used to arrive at an index of test precision called the Difference Two-Sigma Limits as described in ASTM Recommended Practice E 177. This index represents a difference between two test results, from the same test portion, likely to be exceeded only 5% of the time. Thus, if a retest of the same test portion differs from the original test by more than the Difference Two-Sigma Limits, both test results must be held suspect.

Implementation of this type of clearly defined test tolerance, in conjunction with a knowledge of materials and sampling variability, should improve our procedures for design and construction control of highway materials. It should also help those charged with administering the test method and sampling procedures to pinpoint the critical areas, if any, where refinement or greater effort is needed. For instance, if testing variance is shown to be small in relation to materials variance, it may be decided that increased sampling is necessary for a better estimate of material quality. A greater number of samples, chosen at random throughout the population of material, will assure a more accurate measurement of the quality of the material as a whole. If, however, testing variance is large in proportion to materials variance, increased sampling would have less effect on reducing the error of the final result than would increased testing of each sample. Thus, the more tests taken of each sample, the less the testing error (variance) contributes to the uncertainty of the final result.

In either case, this report provides some of the tools necessary to make optimum use of manpower in such instances.

CONCLUSIONS

All conclusions reached herein rest on assumptions concerning the physical distribution and consistency of the data. Histograms for each sample graphically illustrated the normality of the data; thus normal statistical analyses were used. Reasonably close agreement on test precision with a prior Sand Equivalent study indicates consistency and stability in the precision results. These findings help reinforce the validity of the following recommended Precision Statements.

Material and Type Index	Standard Deviation	Difference Two-Sigma Limits
Single-Operator Precision:		
S.E. Range Below 45	1.37	4
S.E. Range 45 to 65	2.95	8
S.E. Range Above 65	2.07	6
Multilaboratory Precision:		
S.E. Range Below 45	1.70	5
S.E. Range 45 to 65	3.75	11
S.E. Range Above 65	2.65	8

One method of reducing the large variation in test results in the 45 to 65 range would be to perform two tests on the same sample and report their average. This follows from the statistical principle that says a distribution of averages computed from N individual results is related to the corresponding distribution of individual results by: $\sigma_{avg} = \sigma_{ind}/\sqrt{N}$. Then the Precision Statement would read:

Material and Type Index	Standard Deviation	Difference Two-Sigma Limits
Single-Operator Precision:		
S.E. Range 45 to 65	2.08	6
Multilaboratory Precision:		
S.E. Range 45 to 65	2.65	8

It is a statistical fact that the precision of a particular test is not measureable until a duplicate test is made. To get the most practical benefit from the values of repeatability and reproducibility for the Sand Equivalent Test contained in this report the field engineer must have two test results from the same sample. Thus, the best way of handling a disputed test result is to take a large enough initial sample to permit a retest and then compare these two tests with the Acceptable Range of Two Results (Precision). If their difference is greater than this value call for a third referee test. Otherwise, average the two results and accept or reject on that basis.

Additional statistical analyses of the data reveals other important facts concerning the Sand Equivalent Test. Duncan's Multiple Range Test shows insignificant differences in test precision between lab and field personnel and Idaho and

Mechanical shaker methods. Thus, a single precision statement embracing all these variables may be written. Examination of the precision statements indicates that significant differences in test results according to Sand Equivalent range do exist. This was also demonstrated by performing Bartlett's F-Test [5] on each Sand Equivalent grouping. This is why different precision statements for various ranges of Sand Equivalent values have been provided. Such qualifications can be expected in precision statements for other tests since precision and material type or range are closely related.

RECOMMENDATIONS

The findings of this report combined with information regarding sampling and materials variance can be used to 1) include Precision Statements in Test Method No. Calif. 217, 2) determine if larger or more samples would be justified in order to provide better precision, and 3) determine the overall variabilities of materials with sand equivalent specifications, including the contribution of testing variance, and make any appropriate modification to the specification limits and testing control procedures.

DESCRIPTION OF WORK

Nine samples of minus #4 aggregate secured from highway construction sources throughout the State of California were studied in this experiment. These samples consisted of three different construction materials used in the State. Three samples (Nos. 4, 7 and 8) came from aggregate used as subbase material with Sand Equivalent values greater than 18 (as specified in the Standard Specifications). Three more samples (Nos. 3, 5 and 6) came from aggregate used in asphalt concrete with Sand Equivalent values greater than 48. The remaining three samples (Nos. 1, 2 and 9) were obtained from aggregates used in portland cement concrete with Sand Equivalent values greater than 73. A total of 4,968 Sand Equivalent Tests were made during this study.

In addition to the series of tests conducted at the Transportation Laboratory, tests were conducted at the District Laboratory and two field laboratories in each of five Districts. The Transportation Laboratory and the District Laboratories each had three operators determine Sand Equivalent Test results using both the Mechanical and Idaho Shaker. Each field laboratory had two operators run the test using the Idaho Shaker only.

Each sample to be tested was split by the Transportation Laboratory so that each of the 28 operators would receive a sufficient amount of material to perform 12 tests with the Idaho and 12 with the Mechanical Shaker. The splitting procedure yielded 32 nearly identical sub-samples for each of the 9 samples. These were randomly assigned to the 28 operators with the 4 remaining sub-samples kept for contingency purposes. Each operator verified that his equipment complied with the requirements of Test Method No. Calif. 217. The measure tin used in the test was supplied by the Transportation Laboratory. The operator marked for identification all the samples assigned to him. He then split the sample into eight equal portions as shown in Figure 1 down to level 'A'. These portions were identified and numbered using a random number scheme. Each was then sealed and stored until it was needed.

On the day before testing, one of the eight portions was taken in sequence of its randomly assigned number, adjusted to equal the weight of four tin measures, split into four equal portions and oven dried as described in Test Method No. Calif. 217. Three of the four test portions were tested. The fourth test portion was saved and later blended with other extra portions for sieve analysis. At some test locations temperature control was a problem. About 20% of the tests were run at temperatures outside of the 72 ± 5°F specified by the test method. However, it was felt that this approximated field testing conditions.

Each test portion was weighed to the nearest gram after oven drying. This weight was recorded along with the readings of the top of clay and the top of sand columns (estimated to the nearest hundredth). The above procedures were repeated for three other runs using a total of four of the original eight portions for each of the shaking methods. (Note: Field labs used four of the eight portions since they had only one shaking method available.) The extra four test portions were combined into one sample and a sieve analysis run. The results of the sieve analysis were also recorded on the data sheet (Figure 2).

When the data computation forms were completed, they were sent to the Transportation Laboratory for analysis.

STATISTICAL ANALYSES

The data was analyzed on a General Electric time-share computer terminal located at the Transportation Laboratory. Except for the GE Library Program, "Analysis of Variance", all the programs used for the analyses were written by Transportation Laboratory personnel.

The first step in the overall analysis was determining each individual operator's mean and standard deviation for each of the nine samples (12 tests per sample). After this was done, the mean for each group of operators within each

CHART
SPLITTING AND RANDOM NUMBERS PLAN
(Each Operator)

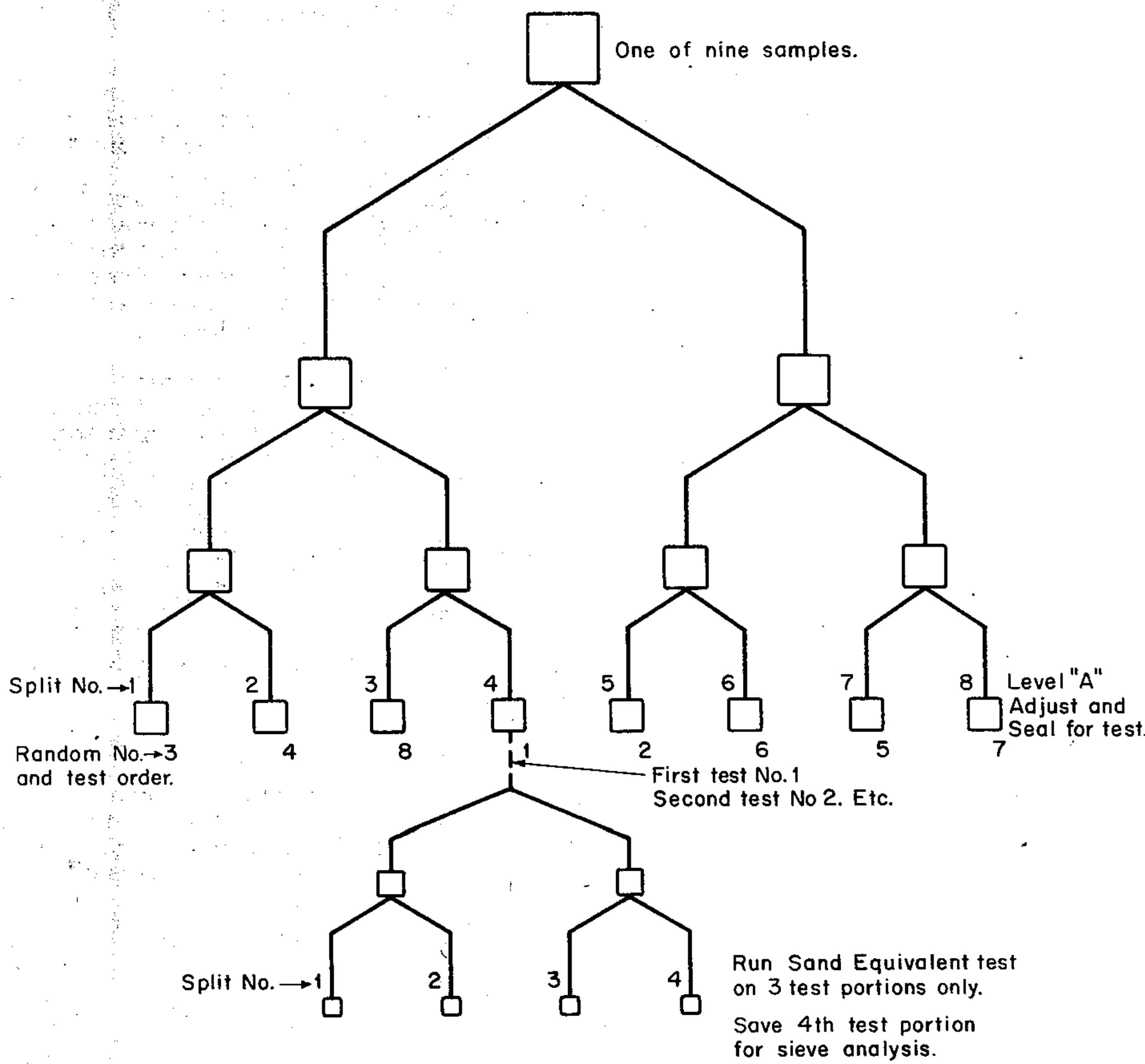

Figure 2

SAND EQUIVALENT TEST DATA FOR PRECISION STATEMENT

DISTRICT 05
Sample Ticket Code 70-1501
Operator Name J. SMITH

Random No.		DATE	ROOM TEMP °F	TEST NO.	OVEN DRY WT.	READINGS (to .01") TOP CLAY	TOP SAND	REMARKS
1	MECHANICAL SHAKER	8-20-70	79°	1	137	5.49	4.06	ADDED TOO MUCH H_2O PRIOR TO SPLIT
				2	140	5.70	4.16	
				3	144	5.90	4.30	
8		8-21-70	77°	4	143	5.71	4.30	
				5	138	5.43	4.18	
				6	143	5.41	4.19	
4		8-24-70	79°	7	143	5.71	4.32	
				8	140	5.68	4.20	
				9	142	5.62	4.12	
3		8-25-70	78°	10	138	5.60	4.16	SCOOPED, TOO WET TO SPLIT
				11	135	5.43	4.02	
				12	141	5.78	4.18	
7	IDAHO SHAKER	8-20-70	79°	1	142	5.82	4.16	IDAHO CLAMPED TO BENCH WHICH VIBRATED SLIGHTLY
				2	140	5.11	4.18	
				3	137	5.42	4.09	
5		8-21-70	77°	4	134	5.41	4.10	
				5	131	5.28	3.93	
				6	129	5.22	3.88	
6		8-24-70	79°	7	143	5.83	4.12	
				8	140	5.74	4.19	
				9	140	5.66	4.10	
2		8-25-70	78°	10	141	5.63	4.03	
				11	134	5.40	4.11	
				12	155	6.22	4.52	

Mechanical Shaker:

SEIVE SIZE	PASSING %
3/8"	
#4	100
#8	81
#16	58
#30	39
#50	18
#100	7
#200	5

Idaho Shaker:

SEIVE SIZE	PASSING %
3/8"	
#4	100
#8	80
#16	56
#30	37
#50	17
#100	7
#200	4

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district and the standard deviation from this district mean were calculated. The same procedure was used in calculating the mean and standard deviation for all the operators on a statewide basis. Table 1 lists all the individual, district-wide and statewide means and standard deviations determined for this study.

Since the standard deviations for all the operators had been calculated, a precision statement could have been determined for each operator. However, there was no accurate representation of the average individual's standard deviation. Also, it was stated in the testing procedure that each operator was to test only three of the twelve samples on any one day (called a run). Thus, it would take at least four days for an operator to complete his twelve tests. Quite often this testing procedure was stretched out over a two week period. Therefore, there was no estimate of the basic error variance of replicate tests done on the same day.

These considerations led to the use of Analysis of Variance [4] (ANOVA). ANOVA techniques allowed isolation of the single-operator variance on a one day basis. This term, called the replicate or error variance, was established by treating the data as a whole, and thereby accurately represented the average operator. For purposes of symmetry (needed to apply the ANOVA technique) only data from the district laboratories and the Transportation Laboratory was analyzed. Since the field laboratories ran only one shaker method, a between method variance could not be determined using their data. Duncan's Multiple Range Test, however, showed no significant difference between field and lab results thereby justifying omission of the field data from the ANOVA (see page 8).

The expected mean squares (Table 7, Appendix B) were used to isolate all other systematized sources of variance in the test data: runs, methods, operators, districts and samples. Each of these results represented the variance due exclusively to the particular source. Therefore, the total multilaboratory variance was an additive combination of the error, run, operator and district variances (see Tables 2, 3 and 4). The variance between the Idaho and Mechanical shaker was not included in this sum because it was proven to be insignificant by an F-Ratio Test [5] (see page 9).

Since it had been suspected that test precision varied with the Sand Equivalent Value, each type of aggregate was run as a separate ANOVA. These suspicions were confirmed later by the use of an F-Ratio Test on the single-operator variances for

TABLE 1

MEAN AND STANDARD DEVIATION

IDAHO AND MECHANICAL METHODS

SOIL SAMPLE			PCC 1		PCC 2		AC 3		BASE 4		AC 5		AC 6		BASE 7		BASE 8		PCC 9	
DIST	OPR	$\bar{x}$ σ	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH
DISTRICT A	1	$\bar{x}$	80.96	80.34	76.41	76.24	57.58	56.03	25.44	24.94	57.00	56.93	57.22	54.70	29.70	28.45	26.93	25.66	83.81	82.46
		σ	1.66	1.44	1.22	0.67	1.78	2.75	1.11	0.92	1.85	1.90	2.28	1.50	0.94	0.36	0.82	0.78	1.58	0.96
	2	$\bar{x}$	82.07	80.18	76.18	75.11	54.85	56.19	24.90	24.66	55.39	55.80	54.49	54.92	28.13	27.69	25.74	25.37	80.18	80.63
		σ	2.54	2.02	1.90	2.22	2.61	2.66	1.76	1.86	3.05	2.94	2.53	2.35	0.68	0.49	1.07	0.52	1.88	2.23
	3	$\bar{x}$	80.18	79.55	74.32	74.17	55.30	56.06	24.44	24.93	55.42	55.91	56.74	57.71	29.63	29.06	26.47	25.90	80.36	80.57
		σ	1.41	1.22	1.37	0.99	1.98	0.94	0.73	0.61	2.20	2.07	1.48	1.48	1.10	0.45	0.90	0.69	1.06	1.07
	4	$\bar{x}$	80.41		72.63		53.96		24.98		59.76		57.57		28.33		25.83		82.52	
		σ	1.10		1.58		1.01		0.70		0.94		1.38		0.80		0.73		0.67	
	5	$\bar{x}$	83.17		76.07		61.24		26.77		63.75		61.67		31.16		27.78		83.77	
		σ	1.93		1.37		2.11		1.14		1.39		2.23		0.55		0.89		1.56	
DISTRICT-WIDE		$\bar{x}$	81.36	80.02	75.12	75.17	56.58	56.09	25.31	24.84	58.27	56.21	57.54	55.77	29.39	28.40	26.55	25.64	82.13	81.22
		σ	2.07	1.59	2.06	1.65	3.25	2.21	1.38	1.22	3.74	2.34	3.06	2.25	1.37	0.71	1.14	0.69	2.10	1.73
DISTRICT B	1	$\bar{x}$	80.61	80.18	74.99	73.72	58.32	56.99	22.84	22.92	57.25	57.45	57.31	58.21	27.33	27.76	24.72	24.98	82.18	83.15
		σ	1.29	1.19	1.91	1.36	3.93	2.52	0.63	0.62	1.82	2.05	3.35	3.19	0.48	0.54	0.61	0.53	0.97	2.03
	2	$\bar{x}$	84.31	80.02	78.08	74.80	58.22	56.87	24.69	24.30	60.61	55.63	59.57	56.73	29.67	27.89	26.20	25.40	83.55	82.56
		σ	1.47	1.66	1.63	1.23	1.54	1.62	0.81	1.09	2.39	1.57	2.44	1.88	1.03	0.63	0.70	0.55	0.98	0.99
	3	$\bar{x}$	79.41	79.66	75.57	76.26	56.13	55.73	25.36	25.37	57.32	60.26	58.03	59.57	27.94	27.54	24.98	25.15	80.49	80.87
		σ	1.86	1.71	0.73	5.34	3.65	2.87	1.01	1.40	2.29	6.07	3.27	1.43	1.07	0.79	0.79	0.82	1.33	1.53
	4	$\bar{x}$	77.02		73.53		54.05		24.05		51.51		55.75		27.50		25.64		80.79	
		σ	1.36		0.85		2.58		1.33		2.72		1.55		1.34		0.97		1.81	
	5	$\bar{x}$	84.37		82.78		64.77		25.60		58.40		60.30		30.30		26.44		85.98	
		σ	1.34		1.89		1.84		0.61		2.50		2.08		0.49		0.47		0.95	
DISTRICT-WIDE		$\bar{x}$	81.14	79.95	76.99	74.93	58.30	56.53	24.51	24.20	57.02	57.78	58.19	58.17	28.55	27.73	25.59	25.18	82.58	82.19
		σ	3.21	1.51	3.53	3.33	4.56	2.40	1.34	1.46	3.80	4.17	3.02	2.52	1.52	0.66	0.97	0.65	2.36	1.81

$\bar{x}$ = MEAN OF 12 TESTS

TABLE 1 (continued)

SOIL SAMPLE			PCC 1		PCC 2		AC 3		BASE 4		AC 5		AC 6		BASE 7		BASE 8		PCC 9	
DIST	OPR	x̄ σ	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH	IDAHO	MECH
DISTRICT C	1	x̄	79.99	78.23	74.14	74.42	57.85	59.25	23.30	25.54	53.62	57.86	57.02	62.44	27.58	28.59	25.15	26.24	80.61	80.62
		σ	3.43	2.09	3.01	1.61	4.02	3.28	1.95	0.86	2.07	1.40	2.53	3.77	0.85	0.93	1.27	1.34	5.48	0.85
	2	x̄	77.48	77.70	73.66	72.07	51.27	53.84	22.18	23.44	53.19	55.69	57.14	58.84	24.87	26.42	23.91	24.22	76.64	78.55
		σ	2.87	1.19	5.20	0.99	1.24	1.84	1.11	0.35	2.00	1.09	2.63	1.57	1.44	0.95	1.46	0.62	1.99	1.53
	3	x̄	80.17	78.06	74.20	73.42	54.68	53.93	25.22	24.92	58.63	57.08	61.44	60.53	29.19	28.49	26.08	25.95	82.22	81.61
		σ	1.85	1.08	1.72	1.70	3.42	2.76	1.08	0.54	2.42	1.35	3.02	1.07	0.83	0.36	0.59	0.81	0.71	0.75
	4	x̄	81.14		76.54		57.85		28.11		60.15		56.61		29.87		26.38		83.86	
		σ	2.46		3.16		2.90		2.55		1.40		3.18		1.25		1.07		1.36	
	5	x̄	80.21		75.22		57.32		23.38		58.76		61.02		29.10		24.09		82.18	
		σ	1.23		2.10		4.71		1.17		2.71		1.93		0.81		1.05		0.83	
DISTRICT-WIDE		x̄	79.80	77.99	74.75	73.30	55.79	55.67	24.44	24.63	56.87	56.88	58.64	60.60	28.12	27.83	25.12	25.47	81.10	80.26
		σ	2.70	1.49	3.32	1.73	4.22	3.66	2.65	1.08	3.58	1.55	3.37	2.80	2.08	1.27	1.48	1.30	3.61	1.68
DISTRICT D	1	x̄	81.74	83.27	76.19	77.62	61.36	62.64	24.71	25.59	61.60	65.15	60.47	61.87	30.09	30.10	23.56	25.06	83.95	85.23
		σ	2.43	2.36	1.56	1.44	5.93	3.78	2.38	1.49	1.10	2.15	2.19	2.33	1.11	1.63	1.56	1.26	0.86	1.26
	2	x̄	82.10	81.47	76.72	75.49	58.97	58.98	25.93	25.25	64.19	62.60	60.38	60.21	30.00	29.17	25.94	25.67	83.79	81.28
		σ	2.20	2.13	1.77	2.30	3.49	4.18	1.40	1.23	2.00	2.19	3.76	3.90	1.94	2.33	1.34	1.14	2.94	1.45
	3	x̄	83.42	82.53	78.78	75.46	56.20	54.57	27.23	25.89	64.85	60.73	63.84	59.01	31.81	27.00	25.53	24.33	85.46	82.57
		σ	3.29	2.53	3.85	1.22	1.70	2.06	1.75	1.58	1.31	1.54	2.25	2.41	0.92	0.65	1.30	0.78	1.70	1.03
	4	x̄	80.31		75.24		60.22		24.80		59.21		59.33		29.06		24.08		83.14	
		σ	1.02		0.99		2.44		0.89		1.45		1.29		0.45		0.79		1.04	
	5	x̄	82.23		76.14		60.68		26.05		58.45		60.19		28.84		27.49		82.66	
		σ	1.32		1.40		1.55		0.41		1.63		2.19		1.51		0.86		1.22	
DISTRICT-WIDE		x̄	81.96	82.42	76.61	76.19	59.49	58.73	25.74	25.58	61.66	62.83	60.84	60.36	29.96	28.76	25.32	25.02	83.80	83.02
		σ	2.36	2.40	2.40	1.96	3.78	4.74	1.75	1.42	2.98	2.66	2.85	3.12	1.63	2.10	1.83	1.19	1.92	2.07

x̄ = MEAN OF 12 TESTS

TABLE 1 (continued)

SOIL SAMPLE DIST	OPR	$\bar{x}$ σ	PCC 1 IDAHO	PCC 1 MECH	PCC 2 IDAHO	PCC 2 MECH	AC 3 IDAHO	AC 3 MECH	BASE 4 IDAHO	BASE 4 MECH	AC 5 IDAHO	AC 5 MECH	AC 6 IDAHO	AC 6 MECH	BASE 7 IDAHO	BASE 7 MECH	BASE 8 IDAHO	BASE 8 MECH	PCC 9 IDAHO	PCC 9 MECH
DISTRICT	1	$\bar{x}$	76.53	80.85	73.43	76.51	56.83	56.50	24.88	27.19	56.42	56.96	57.04	57.92	29.36	29.37	25.55	27.04	79.96	82.26
		σ	1.20	1.22	1.39	0.71	2.39	1.99	1.36	0.90	1.36	1.01	1.45	3.06	0.88	0.69	0.35	0.74	0.92	1.48
	2	$\bar{x}$	80.30	82.02	74.86	77.31	56.18	59.05	25.40	28.55	53.04	57.47	54.30	57.50	29.37	30.91	27.37	28.61	79.60	82.12
		σ	1.14	1.14	1.42	0.76	1.14	1.36	1.51	1.66	2.19	1.73	3.31	1.74	0.45	0.72	0.82	0.84	1.14	1.29
	3	$\bar{x}$	79.50	81.11	76.81	76.10	60.18	58.27	25.82	27.48	56.28	57.55	59.08	57.15	28.88	29.03	26.13	27.73	80.37	81.77
		σ	1.57	1.02	0.87	2.49	2.95	1.88	0.74	0.78	1.35	1.64	0.92	1.12	1.02	1.35	0.81	0.96	2.26	1.02
	4	$\bar{x}$	79.94		73.66		55.39		26.29		54.94		56.00		28.43		25.65		81.21	
		σ	1.56		1.20		0.71		1.70		1.03		2.11		0.43		0.77		0.83	
	5	$\bar{x}$	83.16		74.70		61.37		27.40		59.16		57.65		30.61		25.72		81.97	
		σ	1.64		1.48		5.78		1.83		1.91		2.55		1.39		1.38		0.91	
DISTRICT-WIDE		$\bar{x}$	79.88	81.33	74.69	76.64	57.99	57.94	25.96	27.74	55.97	57.33	56.81	57.52	29.33	29.77	26.08	27.79	80.62	82.05
		σ	2.54	1.21	1.74	1.59	3.85	2.03	1.67	1.29	2.56	1.48	2.69	2.09	1.14	1.25	1.09	1.05	1.55	1.25
D. O. T. LAB	1	$\bar{x}$	84.01	82.74	78.08	77.64	59.86	59.90	27.85	27.66	62.49	60.72	62.16	60.45	29.66	29.14	26.66	26.35	84.74	84.76
		σ	0.88	1.10	0.88	0.80	1.45	0.92	1.16	0.86	1.81	0.76	0.84	1.06	0.74	0.50	0.67	0.69	1.27	0.83
	2	$\bar{x}$	82.53	82.18	77.38	78.27	59.28	59.11	23.61	24.83	59.53	59.35	59.07	58.38	28.62	28.61	24.36	25.39	82.97	84.31
		σ	1.05	1.01	0.99	1.06	1.53	2.36	0.72	0.63	1.27	1.80	1.27	2.40	0.51	0.65	0.43	1.97	2.04	1.08
	3	$\bar{x}$	83.69	83.30	76.51	78.52	62.55	61.07	26.88	29.80	61.86	63.02	62.64	66.03	29.85	29.18	25.69	26.20	84.84	85.59
		σ	1.42	0.48	0.61	1.20	1.77	2.04	1.36	1.25	1.97	1.88	1.77	5.14	0.62	0.39	0.51	0.80	1.28	1.75
DISTRICT-WIDE		$\bar{x}$	83.41	82.75	77.32	78.15	60.56	60.03	26.12	27.43	61.29	61.03	61.29	61.62	29.38	28.98	25.57	25.98	84.18	84.89
		σ	1.28	0.99	1.04	1.07	2.12	2.00	2.13	2.26	2.10	2.16	2.07	4.61	0.82	0.57	1.09	1.33	1.76	1.35
STATE-WIDE		$\bar{x}$	81.11	80.74	75.81	75.73	57.94	57.50	25.29	25.73	58.31	58.68	58.71	59.01	29.10	28.58	25.72	25.85	82.28	82.27
		σ	2.73	2.27	2.78	2.50	4.08	3.36	1.95	2.03	3.88	3.49	3.30	3.60	1.63	1.39	1.40	1.41	2.65	2.20

$\bar{x}$ = MEAN OF 12 TESTS

TABLE 2

SAND EQUIVALENT TEST PRECISION STATEMENTS
SUMMARY OF ANALYSIS OF VARIANCE
(BASE TYPE MATERIAL RANGE BELOW 45)

Variance Source	Variance	Standard Deviation	"Difference Two Sigma Limit" (Maximum acceptable difference between two results of the same material).*
Error (Only)	$\sigma_e^2 = 0.7144$	$\sigma_e = 0.8452$	
Run (Only)	$\sigma_r^2 = 1.1562$	$\sigma_r = 1.0752$	
Operator (Only)	$\sigma_o^2 = 0.6149$	$\sigma_o = 0.7842$	
District (Only)	$\sigma_d^2 = 0.4173$	$\sigma_d = 0.6460$	
Single-Operator Precision	$\sigma_e^2+\sigma_r^2=$ 1.8706	1.37	3.87 Say 4
Multi-Operator Precision	$\sigma_e^2+\sigma_r^2+\sigma_o^2=$ 2.4855	1.58	4.46 Say 4
Multilaboratory Statewide Precision	$\sigma_e^2+\sigma_r^2+\sigma_o^2+\sigma_d^2=$ 2.9028	1.70	4.82 Say 5

* Precision calculated as recommended by ASTM Designation C 670-71T

TABLE 3

SAND EQUIVALENT TEST PRECISION STATEMENTS
SUMMARY OF ANALYSIS OF VARIANCE
(ASPHALT CONCRETE TYPE MATERIAL RANGE 45-65)

Variance Source	Variance	Standard Deviation	"Difference Two Sigma Limit" (Maximum acceptable difference between two results of the same material).*
Error (Only)	$\sigma_e^2 =$ 2.2731	$\sigma_e =$ 1.5077	
Run (Only)	$\sigma_r^2 =$ 6.4456	$\sigma_r =$ 2.5388	
Operator (Only)	$\sigma_o^2 =$ 1.1909	$\sigma_o =$ 1.0913	
District (Only)	$\sigma_d^2 =$ 4.1413	$\sigma_d =$ 2.0350	
Single-Operator Precision	$\sigma_e^2+\sigma_r^2 =$ 8.7187	2.95	8.35 Say 8
Multi-Operator Precision	$\sigma_e^2+\sigma_r^2+\sigma_o^2 =$ 9.9096	3.15	8.92 Say 9
Multilaboratory Statewide Precision	$\sigma_e^2+\sigma_r^2+\sigma_o^2+\sigma_d^2 =$ 14.0509	3.75	10.62 Say 11

* Precision calculated as recommended by ASTM Designation C 670-71T

TABLE 4

SAND EQUIVALENT TEST PRECISION STATEMENTS
SUMMARY OF ANALYSIS OF VARIANCE
(PCC SAND TYPE MATERIAL RANGE ABOVE 65)

Variance Source	Variance	Standard Deviation	"Difference Two Sigma Limit" (Maximum acceptable difference between two results of the same material). *
Error (Only)	$\sigma_e^2 = 2.1154$	$\sigma_e = 1.4544$	
Run (Only)	$\sigma_r^2 = 2.1592$	$\sigma_r = 1.4694$	
Operator (Only)	$\sigma_o^2 = 0.6089$	$\sigma_o = 0.7803$	
District (Only)	$\sigma_d^2 = 2.1474$	$\sigma_d = 1.4654$	
Single-Operator Precision	$\sigma_e^2+\sigma_r^2 =$ 4.2746	2.07	5.86 Say 6
Multi-Operator Precision	$\sigma_e^2+\sigma_r^2+\sigma_o^2 =$ 4.8835	2.21	6.26 Say 6
Multilaboratory Statewide Precision	$\sigma_e^2+\sigma_r^2+\sigma_o^2+\sigma_d^2 =$ 7.0309	2.65	7.50 Say 8

* Precision calculated as recommended by ASTM Designation C 670-71T

each aggregate type. The discrepancy is graphically illustrated as a function of the Difference Two-Sigma Limit [1] (D2S, see Appendix A) for both single-operator and multilaboratory conditions in Figure 3. These two plots are not intended as accurate representations of fully logged functions, but only as an illustration of the general relationship between Sand Equivalent Value and test precision. They clearly point out the decrease in test precision in the mid-range of Sand Equivalent Values. While this had been suspected for some time, to our knowledge it had never been quantitatively determined before.

The risks due to testing taken by either the consumer or producer are dependent upon the precision of the test method and the zone of indifference [10] (see Figures 4, 5, and 6). These risks measure the probability of accepting noncompliance material or rejecting good material. In determining these risks common conditions occurring in practice should be assumed. A particular limit that experience has shown yields an adequate level of quality is chosen as the accept-reject point. For the Sand Equivalent Test the moving average limit [9] best satisfied this criterion. The type of test variation most compatible with the moving average limit is the single-operator variance. This represents the test precision of the average single-operator on a day to day basis. This variance is then reduced to the standard deviation of the mean of five tests, estimated by $\sigma' = \frac{\sigma}{\sqrt{5}}$. The resulting standard deviation of the mean defines the shapes of the distribution curves shown in Figures 4, 5 and 6. All that remains is to assign a zone of indifference establishing the location of these curves with respect to the Moving Average Specification.

The zone of indifference is a region bordered by the lowest population mean the consumer is willing to accept at a given risk and by the highest population mean the producer can meet economically. In the case of the Sand Equivalent Test, the former value is simply the Individual Test specification. The producer's value is, however, a function of several variables: aggregate properties, plant characteristics and production techniques. Therefore, since the producer's limiting value could not be established, a specific producer's risk due to testing could not be assigned. Figures 4, 5, and 6 illustrate the concept of consumer and producer testing risks.

Previous studies by the California Transportation Laboratory [7] indicate that materials and sampling variance together usually compose a much greater part of the total variation inherent in test results than does error due to testing. As a result, the actual risks taken by either producer or consumer depend largely upon the materials control exercised by the producer's plant. Ideally, the producer given a minimum target value based on testing risk, should be able to adjust his operations to achieve a low total risk.

Figure 3

CHANGE IN PRECISION VS. CHANGE IN SAND EQUIVALENT VALUE FOR MULTILABORATORY & SINGLE-OPERATOR PRECISION

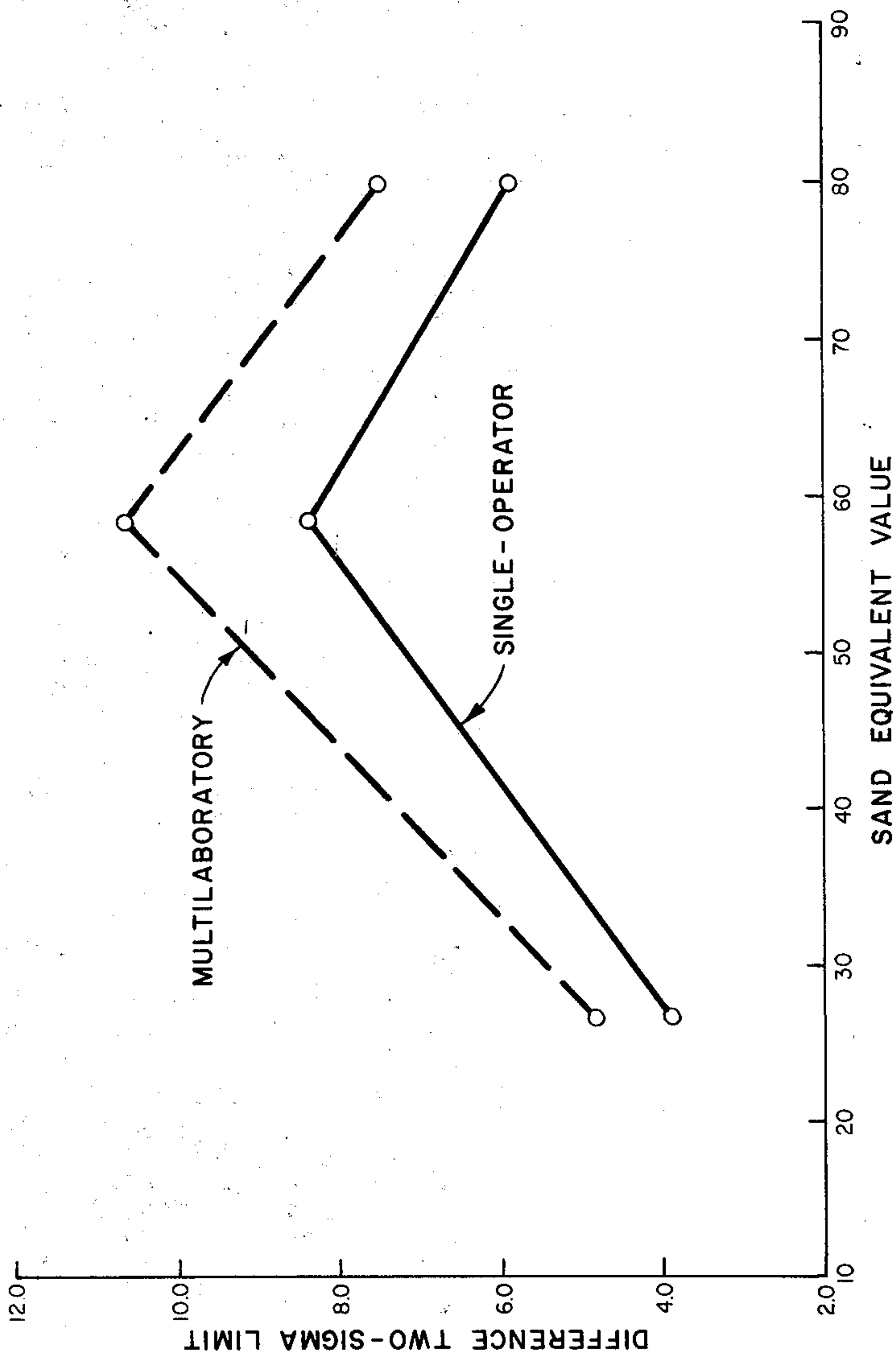


Figure 4

EXAMPLE OF
CONSUMER(β) PRODUCER(α) TESTING RISKS
FOR SUBBASE AGGREGATE

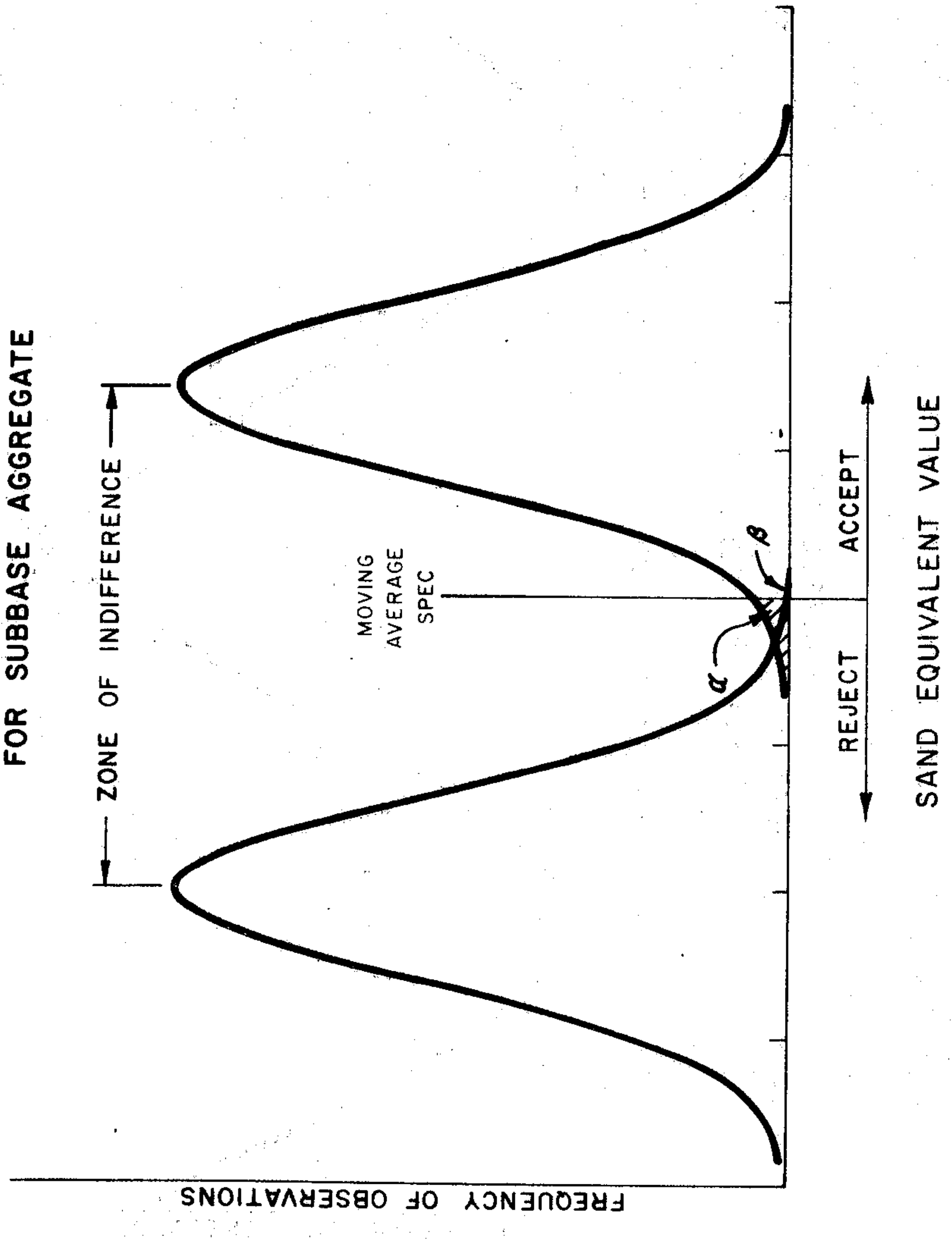


Figure 5

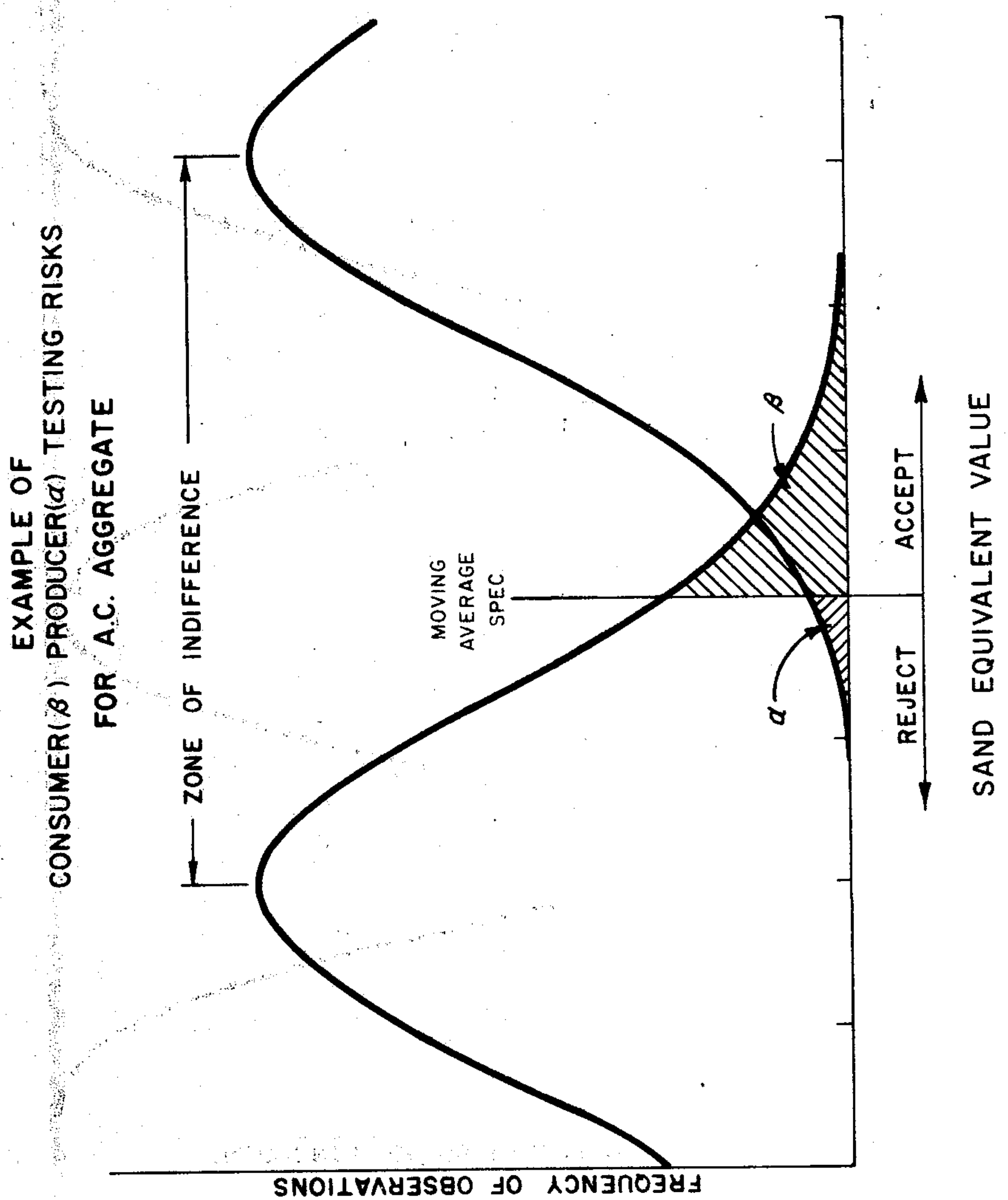
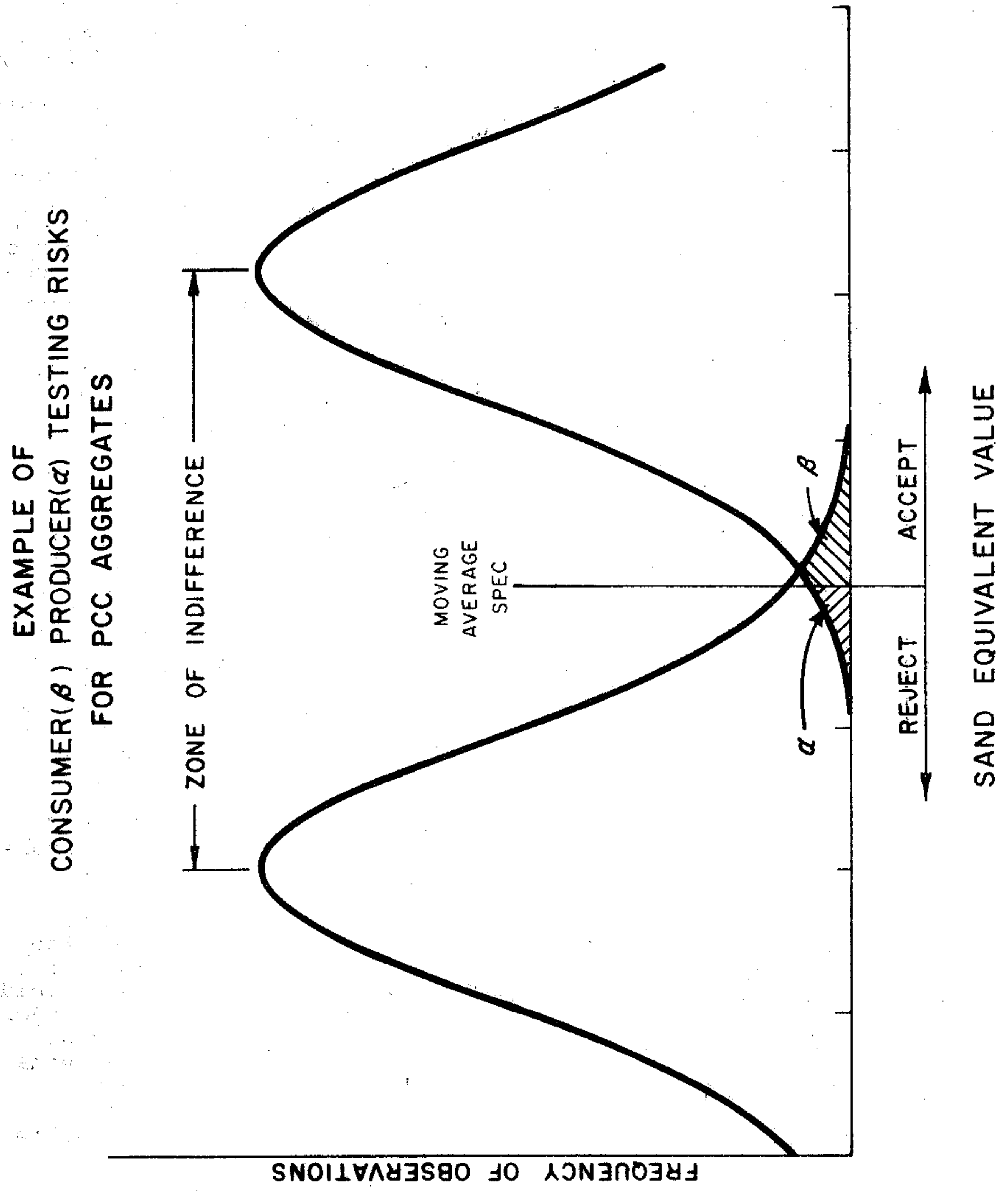

EXAMPLE OF
CONSUMER(β) PRODUCER(α) TESTING RISKS
FOR A.C. AGGREGATE
ZONE OF INDIFFERENCE
MOVING
AVERAGE
SPEC
β
α
REJECT
ACCEPT
SAND EQUIVALENT VALUE
FREQUENCY OF OBSERVATIONS

Figure 6


EXAMPLE OF
CONSUMER(β) PRODUCER(α) TESTING RISKS
FOR PCC AGGREGATES
ZONE OF INDIFFERENCE
MOVING
AVERAGE
SPEC
β
α
ACCEPT
REJECT
SAND EQUIVALENT VALUE
FREQUENCY OF OBSERVATIONS

As a measure of consistency and accuracy, a comparison was made between this Sand Equivalent precision study and an earlier one. The prior study used only three operators, all from the Transportation Laboratory. Each operator ran ten individual Sand Equivalent Tests on three separate fine aggregates (PCC, AC, and AB) by both the Idaho and Mechanical Shaker methods. A total of 180 Sand Equivalent Tests were made. After the results of these tests were obtained, it was felt that there was not enough data collected to adequately determine on a statewide basis a Precision Statement for the Sand Equivalent Test. However, since the Transportation Laboratory operators were the same for each study, and the three soil samples used were similar to three of the nine total samples used in this present study, a comparison could be made between the two studies. The results of this comparison are shown in Table 5. The difference between the means can be attributed to normal sampling and materials variations since the samples were taken out of separate stock piles at different times. Over a year elapsed between the two studies. In comparing the standard deviations between the two studies an F-Ratio Test was used. Of the 18 comparisons, only three were found to be significantly different at the 95 percent confidence level. This indicates the test method was performed consistently over a long period of time.

Each District in the present study had two distinct groups of operators performing the Sand Equivalent Tests. One group of three operators worked in the District Materials Laboratory while the other group of two operators worked in field laboratories located on construction projects within the District. It was decided to compare the means (listed in Table 1) of the field and lab operators for the Idaho Shaker to see if there were any significant differences between the two. To determine this, a computer program was written for Duncan's Multiple Range Test [4] for the significance of means. This test was run for all five operators within each of the five Districts and for all nine samples resulting in a total of forty-five comparisons. A large number of these comparisons (32) showed a significant difference between the means of the five operators. This agrees with the F-Ratio Test in ANOVA which showed the significance of the operator effect. However, there was no consistent pattern developed between field and lab personnel. Table 6 shows that one of the two field men in four out of the five districts obtained the highest Sand Equivalent Value for most of the samples. In only one District, however, was a field man significantly different from the other operators for a majority of the samples. Therefore, it was concluded that there is no significant difference, on a statewide basis, between the means of the field and laboratory operators. (Table 6 is read as follows: For District A, Operator Field 5 had the highest reading in 7 of the 9 samples tested, with 6 of these 7 results

TABLE 5

SAND EQUIVALENT PRECISION STATEMENT

PRIOR S.E. STUDY 1969 VS PRESENT S.E. STUDY 1971

		OPERATOR 1				OPERATOR 2				OPERATOR 3			
		IDAHO		MECH		IDAHO		MECH		IDAHO		MECH	
		Prior	Present	Prior	Present	Prior	Present	Prior	Present	Prior	Present	Prior	Present
SOIL SAMPLE A	$\bar{X}$	80.73	77.38	78.85	78.27	79.51	76.51	79.52	78.52	79.36	78.08	79.27	77.64
	σ	1.06	0.99	1.28	1.06	0.95	0.61	1.19	1.20	1.49	0.88	*1.83	0.80
SOIL SAMPLE B	$\bar{X}$	53.26	59.28	52.18	59.11	54.10	62.55	52.14	61.07	53.19	59.86	52.07	59.90
	σ	1.07	1.53	1.49	2.36	1.28	1.77	1.09	*2.04	1.20	1.45	1.47	0.92
SOIL SAMPLE C	$\bar{X}$	29.40	23.61	28.94	24.83	31.70	26.88	30.92	29.80	31.15	27.85	30.05	27.66
	σ	*1.27	0.72	0.87	0.63	1.22	1.36	0.98	1.25	0.93	1.16	1.00	0.86

*SIGNIFICANT DIFFERENCE BETWEEN PRIOR AND PRESENT TEST AT 95% CONFIDENCE LEVEL.

TABLE 6

RESULTS OF DUNCAN'S MULTIPLE RANGE TEST[1]

DISTRICT	OPERATOR	RANK[2]	NO. EXTREME RESULTS[3]	NO. SIGNIF. EXTREME RESULTS[4]
A	Field 5 Lab 2	Highest Lowest	7 5	6 1
B	Field 5 Field 4	Highest Lowest	8 5	4 4
C	Field 4 Lab 2	Highest Lowest	8 8	1 3
D	Lab 3 Field 4	Highest Lowest	7 8	2 1
E	Field 5 Lab 1	Highest Lowest	6 5	2 1

1. Refer to Table 1.
2. The five operators' results for each sample were placed in order, from the lowest reading to the highest. The operators who ranked highest and lowest the most number of times out of the nine samples tested were chosen for the comparison from each District.
3. The number of times the operator ranked high or low out of the nine samples.
4. Significance determined at 5% level.

being significantly different from the lowest result. Also, Operator Lab 2 had the lowest result 5 times, with only one of them being significantly different from the 2nd highest reading.)

In another analysis, the F-Ratio Test was used to determine if any significant difference in test precision occurred between shaker methods. The test indicated no significant difference between the Mechanical and Idaho Shaker at a 95% confidence level.

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APPENDIX A

Glossary

BARTLETT'S TEST: A test measuring the homogeneity of a large number of variances.

CONSUMER TESTING RISK: The probability of accepting bad material because of faulty test results.

DEGREES OF FREEDOM: The number of independent values that can be assigned to a set of observations.

DIFFERENCE TWO-SIGMA LIMIT (D2S): The greatest difference between two randomly selected test results on the same material that can be expected to occur 95% of the time.

DUNCAN'S MULTIPLE RANGE TEST: A test measuring the homogeneity of a large number of means.

EXPECTED MEAN SQUARE: The long range average value of the mean square in algebraic form.

INTERACTION SOURCE: A source of variance attributed to the combined effect of two or more main sources of variance.

MAIN SOURCE: A source of variance due to one particular effect.

MEAN SQUARE: The variance of a set of means.

MULTILABORATORY PRECISION: A measure of the greatest difference between two test results that would be considered acceptable when properly conducted determinations are made by two different operators in different laboratories on portions of a material that are intended to be as nearly identical as possible.

POPULATION MEAN: The average value of a set or collection of observations having one or more properties in common.

PRODUCER'S TESTING RISK: The probability of rejecting good material because of faulty test results.

REPEATABILITY: A general term refering to the precision of a test method in its most restricted form (in this case, a single operator using the same equipment on the same test portion).

REPLICATE TESTS: Ideally, two or more tests conducted under identical conditions, on identical material and by the same operator.

REPRODUCIBILITY: A general term refering to the precision of a test method in its most broad or relaxed form (in this case, different operators in different laboratories using different equipment on the same test portion).

SIGMA (σ): The Greek letter symbolizing standard deviation.

SINGLE-OPERATOR PRECISION: A measure of the greatest difference between two results that would be considered acceptable when properly conducted repetitive determinations are made on the same material by a competent operator.

STANDARD DEVIATION: The square root of variance.

SUM OF SQUARES: The sum of the squares of the deviations of a set of data from its mean.

VARIANCE: A statistical measure of data dispersion defined as the mean of the squares of the deviation of each data point from the arithmetic mean.

APPENDIX B

Analysis of Variance (ANOVA)

The word ANOVA describes a statistical technique that categorizes data and isolates selected sources of variance. A computer is particularly helpful in performing ANOVA's on large amounts of data. For this study a General Electric Library Program called ANVA5 was used.

The G.E. Library Program is a full factorial, five level ANOVA. It prints out the sum of squares (SS), the degrees of freedom (DF) and the mean squares (MS) for all main and interaction sources of variance.

For each grouping of aggregate type the five main sources of variance isolated were: Between samples, districts, operators, shaker methods, and runs, plus the error or replicate variance. These were tested for significance using the F-Ratio Test on the mean squares. The component variances were then computed using the expected mean square equations (EMS) [4].

Once these variances were determined the standard deviation and the Difference Two-Sigma Limit (D2S) were easily calculated. The D2S has been selected as the appropriate index of precision for use in Precision Statements [1].

The EMS table and ANOVA outputs are listed in Tables 7 through 10.

Table 7

EXPECTED MEAN SQUARES

Source	DF	EMS
Samples	2	$v_e+3v_r+24v_{so}+72v_{sd}+432v_s$
Districts	5	$v_e+3v_r+24v_{so}+72v_o+72v_{sd}+216v_d$
S*D	10	$v_e+3v_r+24v_{so}+72v_{sd}$
Operators	12	$v_e+3v_r+24v_{so}+72v_o$
S*O	24	$v_e+3v_r+24v_{so}$
Methods	1	$v_e+3v_r+12v_{som}+36v_{om}+36v_{sdm}+108v_{dm}+216v_{sm}+648v_m$
S*M	2	$v_e+3v_r+12v_{som}+36v_{sdm}+216v_{sm}$
D*M	5	$v_e+3v_r+12v_{som}+36v_{om}+36v_{sdm}+108v_{dm}$
S*D*M	10	$v_e+3v_r+12v_{som}+36v_{sdm}$
O*M	12	$v_e+3v_r+12v_{som}+36v_{om}$
S*O*M	24	$v_e+3v_r+12v_{som}$
Runs	324	v_e+3v_r
Error	864	v_e
Total	1295	

Note: v symbolizes variance

Table 8

ANOVA FOR LOW S.E. AGGREGATES

Source	SS	DF	MS	F-Ratio
Samples	3016.7	2	1508.35	360.605*
Districts	693.02	5	138.604	2.86034
Operators	581.486	12	48.4572	11.5848*
Methods	10.0379	1	10.0379	2.39978
Runs	1719.15	411	4.18283	5.85505*
Errors	617.239	864	0.714397	
Total	6637.63	1295	5.12559	

Source	Variance	Sigma
Samples	3.48187	1.86598
Districts	0.417346	0.646023
Operators	0.614921	0.784169
Methods	0.009036	0.095056
Runs	1.15615	1.07524
Errors	0.714397	0.84522

*Asterisk indicates significant source of variance.

Table 9

ANOVA FOR MEDIUM S.E. AGGREGATES

Source	SS	DF	MS	F-Ratio
Samples	427.678	2	213.839	9.89541*
Districts	5009.38	5	1001.88	9.33224*
Operators	1288.28	12	107.356	4.96792*
Methods	12.7283	1	12.7283	0.589001
Runs	8881.68	411	21.6099	9.50687*
Errors	1963.95	864	2.27309	
Total	17583.7	1295	13.5781	

Source	Variance	Sigma
Samples	0.444975	0.667064
Districts	4.1413	2.03502
Operators	1.19092	1.09129
Methods	zero	
Runs	6.44561	2.53882
Errors	2.27309	1.50768

*Asterisk indicates significant source of variance.

Table 10

ANOVA FOR HIGH S.E. AGGREGATES

Source	SS	DF	MS	F-Ratio
Samples	9667.31	2	4833.66	562.502*
Districts	2581.39	5	516.277	9.84659*
Operators	629.185	12	52.4321	6.10163*
Methods	1.21289	1	1.21289	0.141147
Runs	3531.78	411	8.59313	4.06222*
Errors	1827.69	864	2.11538	
Total	18238.6	1295	14.0838	

Source	Variance	Sigma
Samples	11.1691	3.34202
Districts	2.14743	1.46541
Operators	0.608874	0.780304
Methods	zero	
Runs	2.15925	1.46944
Errors	2.11538	1.45443

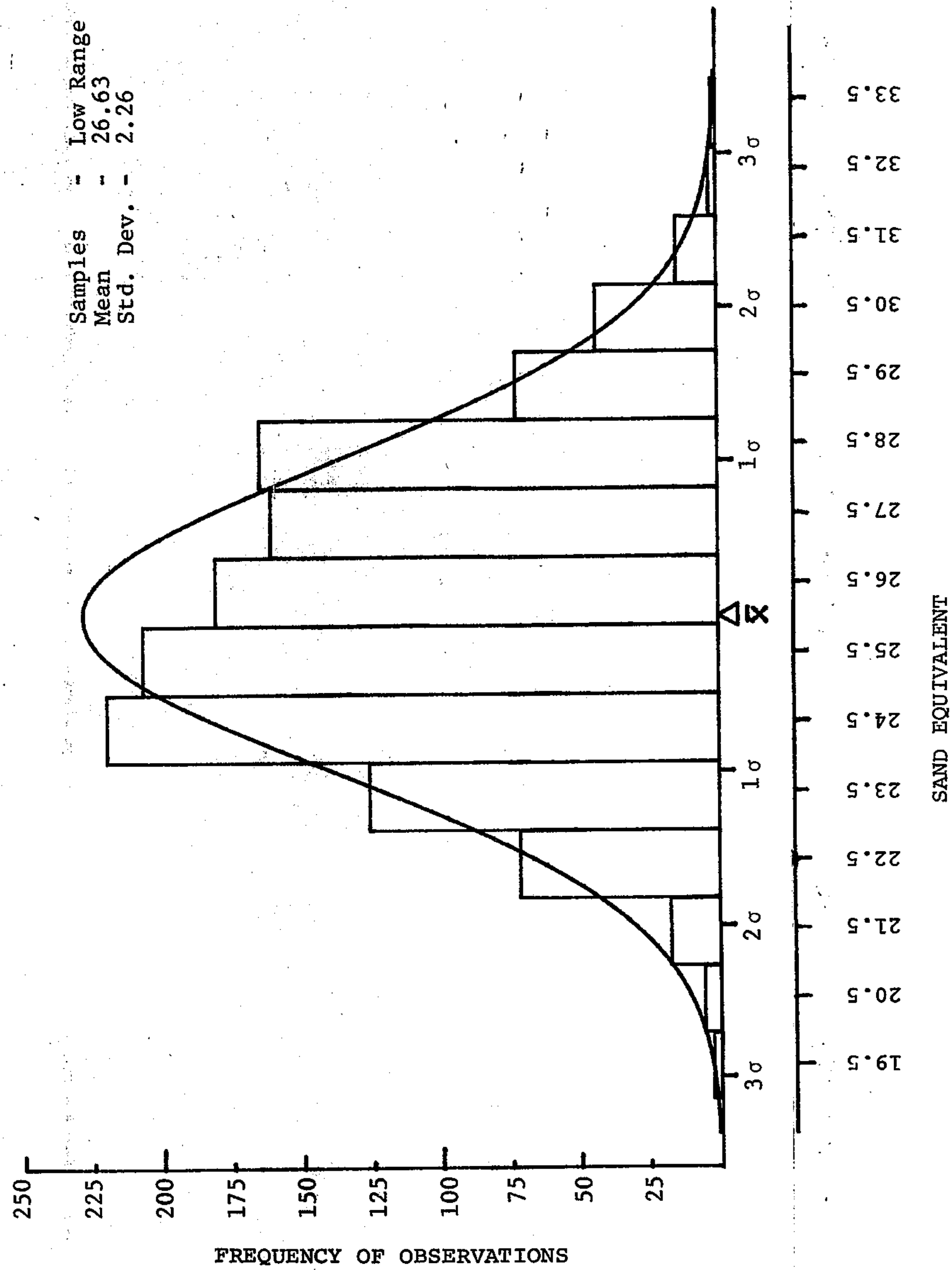
*Asterisk indicates significant source of variance.

APPENDIX C

Histograms

Included are 3 histograms summarizing results for the three material types tested. The histogram data, shown in Figures 8 to 10, was graphed using a Hewlett-Packard plotter in conjunction with the Transportation Laboratory's GE time-share computer. The computer plotted Sand Equivalent Values versus frequency in histogram form, and then superimposed the normal curve computed from the data over the histogram. With this type of arrangement, the dispersion and normality of the Sand Equivalent results can easily be seen.

Figure 7



Samples - Low Range
Mean - 26.63
Std. Dev. - 2.26
FREQUENCY OF OBSERVATIONS
250
225
200
175
150
125
100
75
50
25
SAND EQUIVALENT
19.5
20.5
21.5
22.5
23.5
24.5
25.5
26.5
27.5
28.5
29.5
30.5
31.5
32.5
33.5
3σ
2σ
1σ
$\bar{X}$
1σ
2σ
3σ

Figure 8

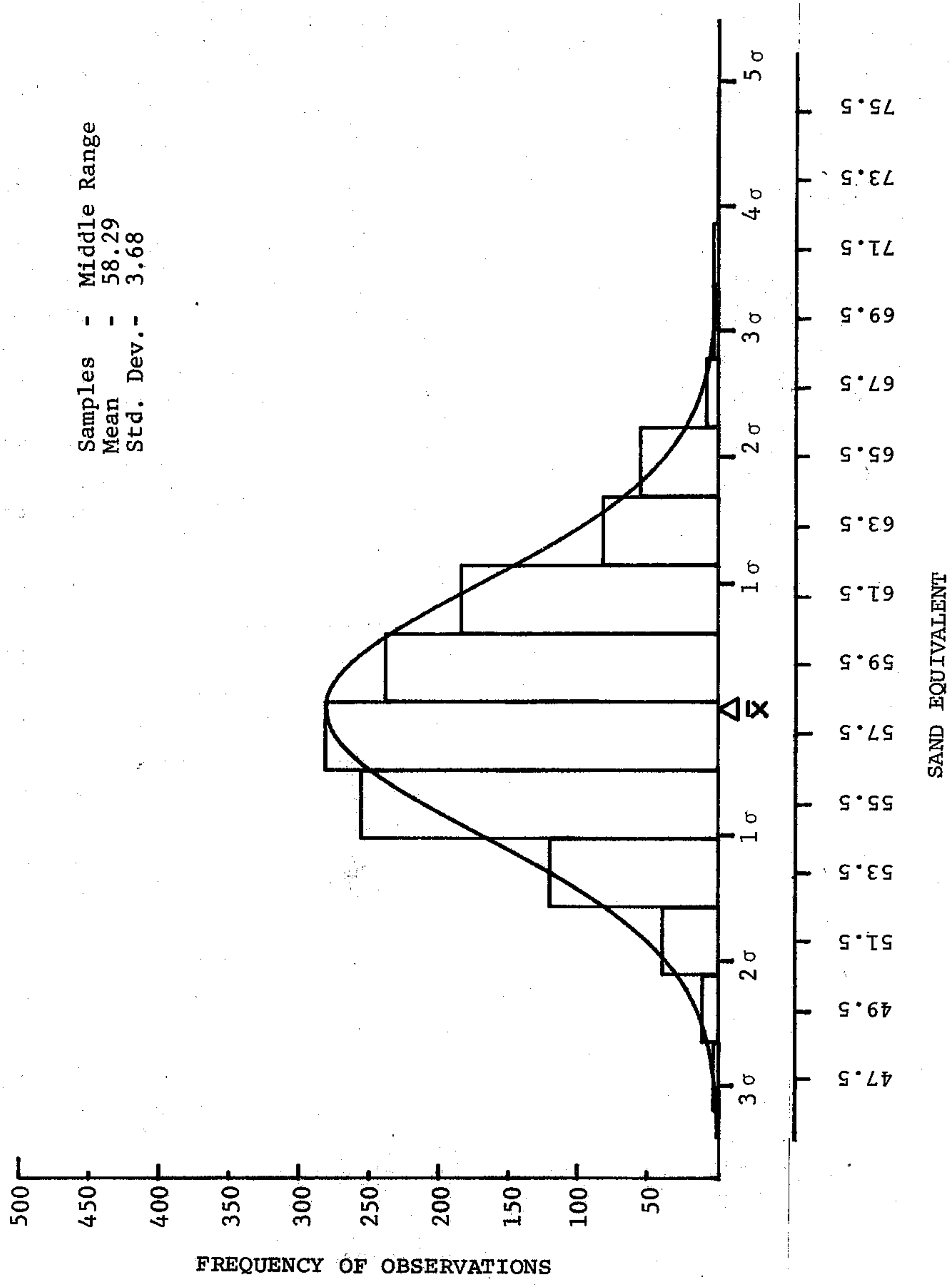
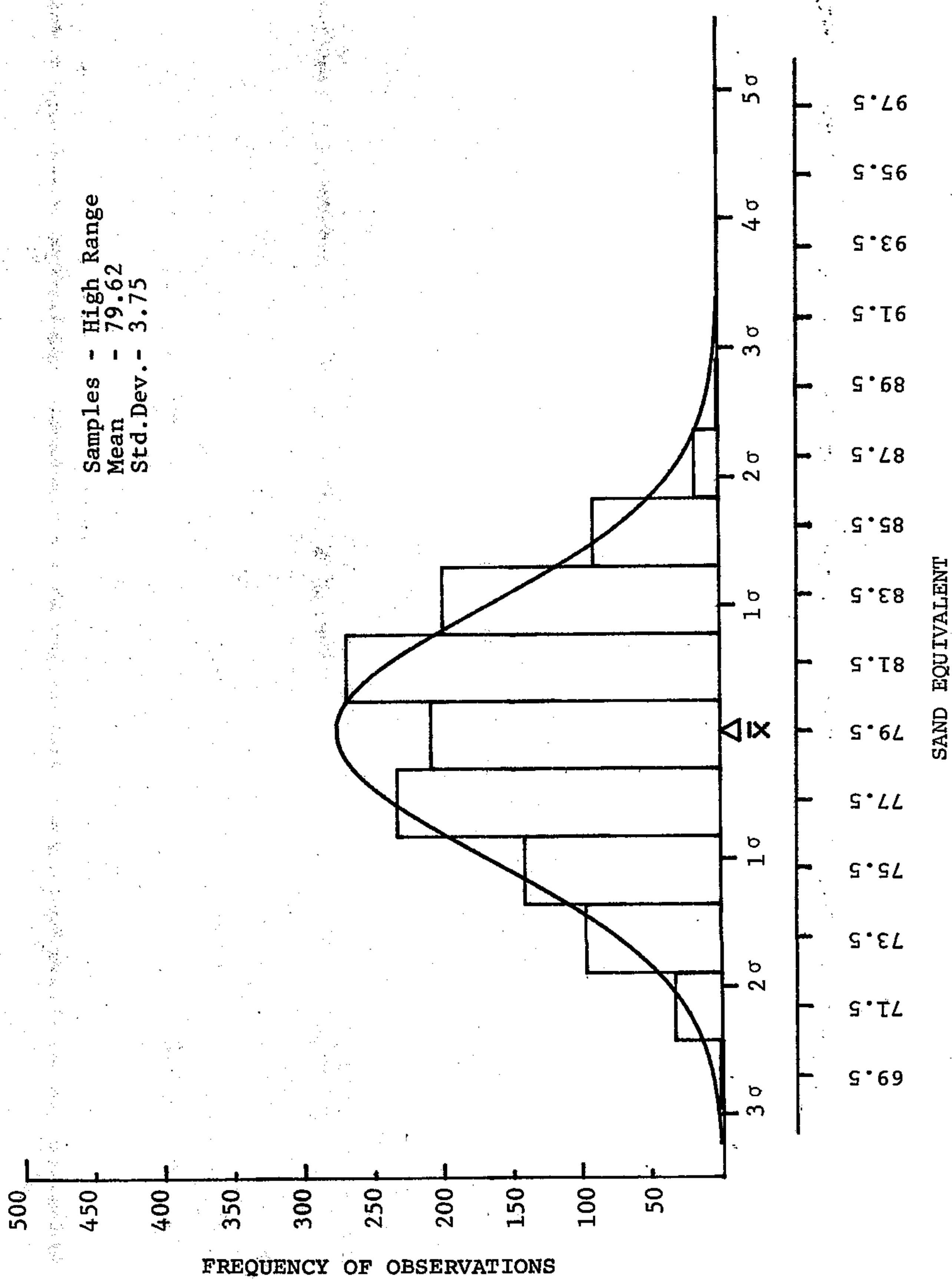

Samples - Middle Range
Mean - 58.29
Std. Dev. - 3.68
FREQUENCY OF OBSERVATIONS
500
450
400
350
300
250
200
150
100
50
SAND EQUIVALENT
47.5
49.5
51.5
53.5
55.5
57.5
59.5
61.5
63.5
65.5
67.5
69.5
71.5
73.5
75.5
3 σ
2 σ
1 σ
x̄
1 σ
2 σ
3 σ
4 σ
5 σ

Figure 9


Samples - High Range
Mean - 79.62
Std.Dev.- 3.75
SAND EQUIVALENT
69.5
71.5
73.5
75.5
77.5
79.5
81.5
83.5
85.5
87.5
89.5
91.5
93.5
95.5
97.5
3σ
2σ
1σ
X̄
1σ
2σ
3σ
4σ
5σ
500
450
400
350
300
250
200
150
100
50
FREQUENCY OF OBSERVATIONS

APPENDIX D

Scatter Diagrams

The following scatter diagrams provide a convenient graphical presentation of the data for six of the nine samples used in the experiment. They were drawn up for one of the three possible sample combinations for each material group. Each of the two shaker methods was graphed separately. The statewide mean and standard deviation for each sample were plotted, dividing the paper into four quadrants.

The upper right quadrant corresponds to a region where points represent values greater than the average for both materials. The lower left quadrant is a region where the points represent values less than the average for both materials. The other two quadrants provide for values one of which is greater than the average and one below the average. If only random errors of the precision type are present in the results, the points may be expected to be distributed equally among the four quadrants. If systematic errors are present, the effect is to shift the points into the upper right or lower left quadrants [11].

If an operator has a systematic error in his testing procedure, his results will generally fall into one quadrant. For instance, the second operator in District C falls into the lower left quadrant on all six scatter diagrams. He consistently obtains low Sand Equivalent Values. This shows that the scatter diagram might prove an effective tool in operator certification programs.

Figure 10

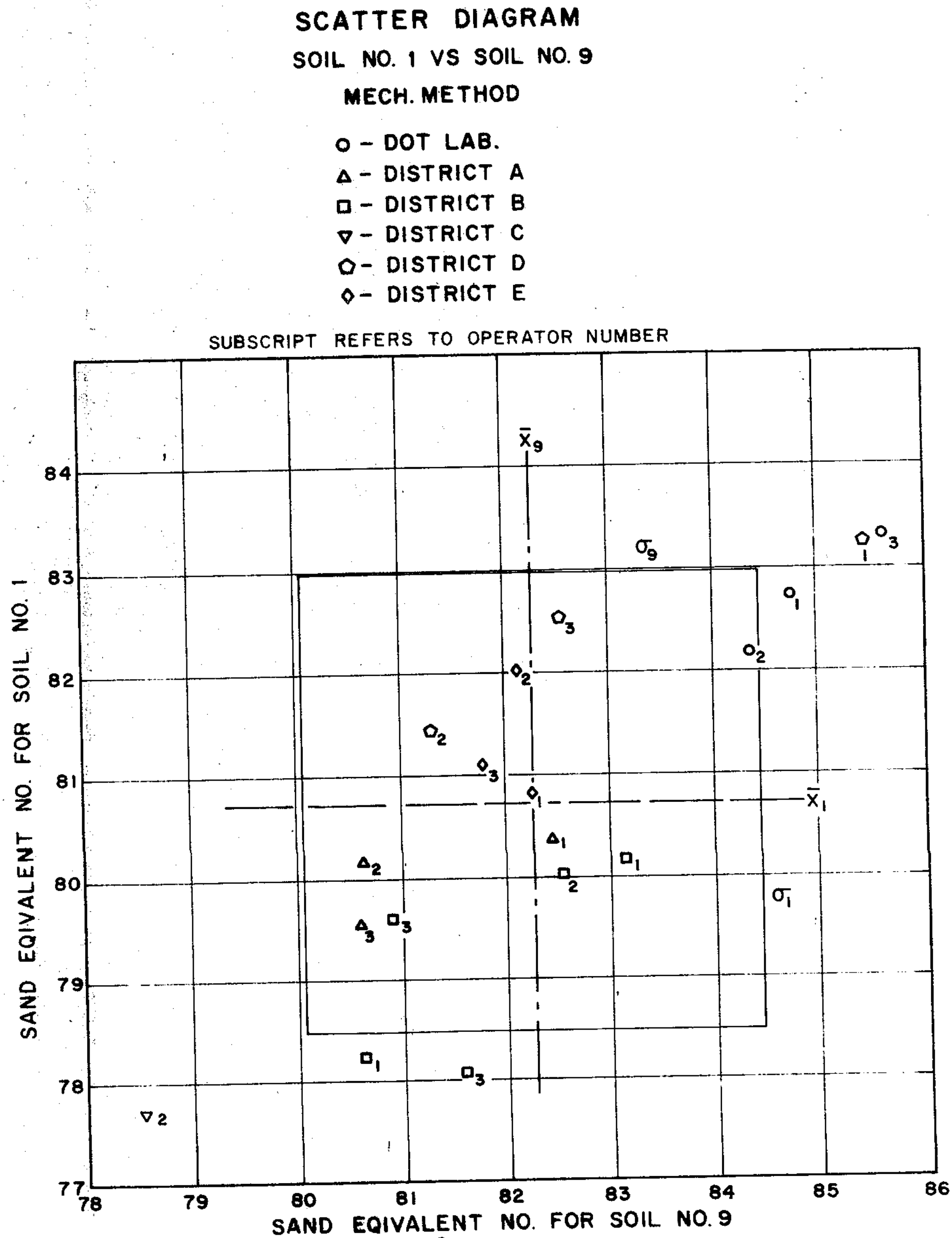

SCATTER DIAGRAM
SOIL NO. 1 VS SOIL NO. 9
MECH. METHOD
○ - DOT LAB.
△ - DISTRICT A
□ - DISTRICT B
▽ - DISTRICT C
⬠ - DISTRICT D
◇ - DISTRICT E
SUBSCRIPT REFERS TO OPERATOR NUMBER
SAND EQIVALENT NO. FOR SOIL NO. 1
SAND EQIVALENT NO. FOR SOIL NO. 9
77
78
79
80
81
82
83
84
85
86
$\bar{X}_9$
$\bar{X}_1$
σ_9
σ_1

Figure 11

SCATTER DIAGRAM

SOIL NO. 1 VS SOIL NO. 9

IDAHO METHOD

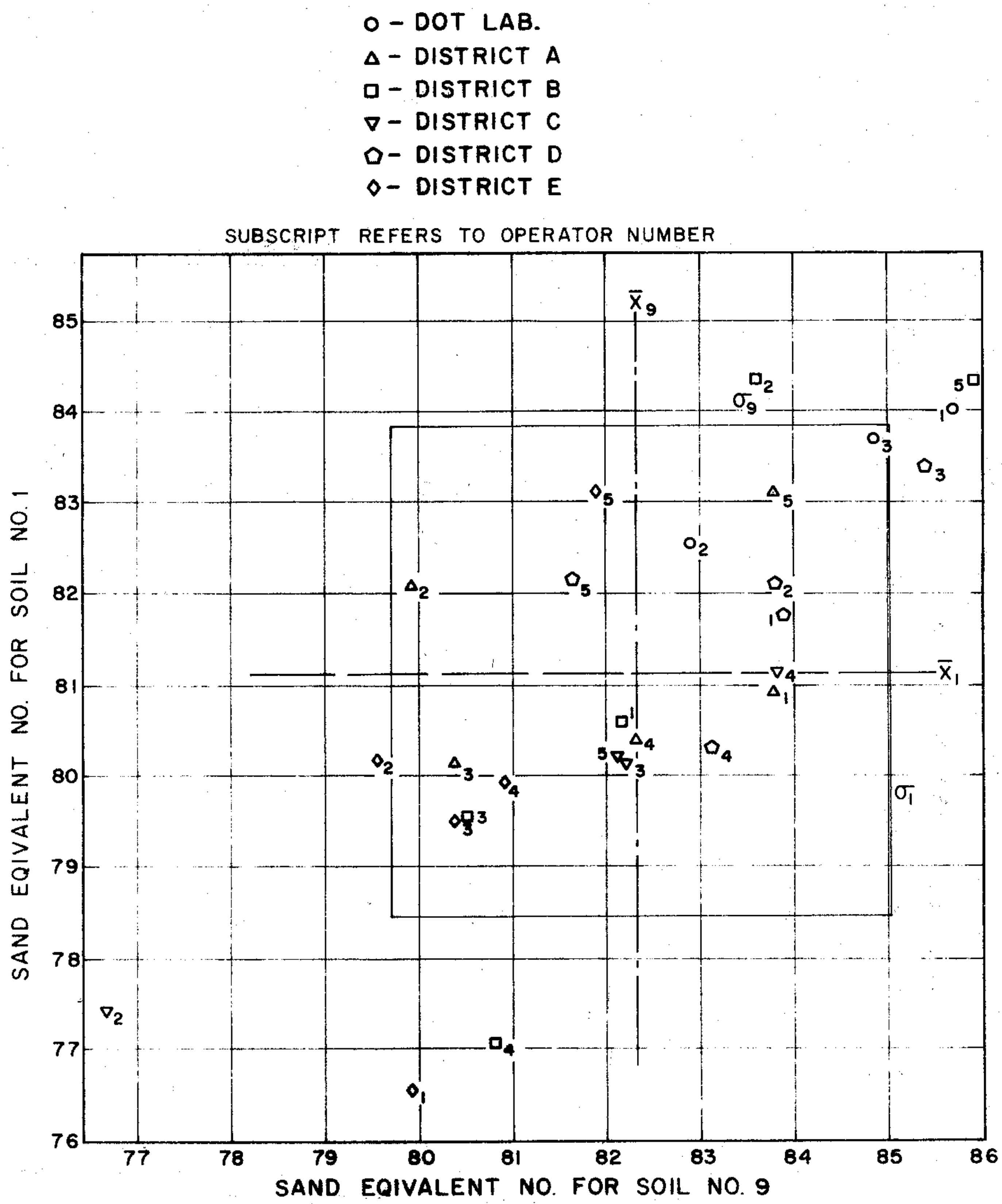


Figure 12

SCATTER DIAGRAM

SOIL NO. 3 VS SOIL NO. 5

MECH. METHOD

o – DOT LAB.
△ – DISTRICT A
□ – DISTRICT B
▽ – DISTRICT C
⬠ – DISTRICT D
◇ – DISTRICT E

SUBSCRIPT REFERS TO OPERATOR NUMBER

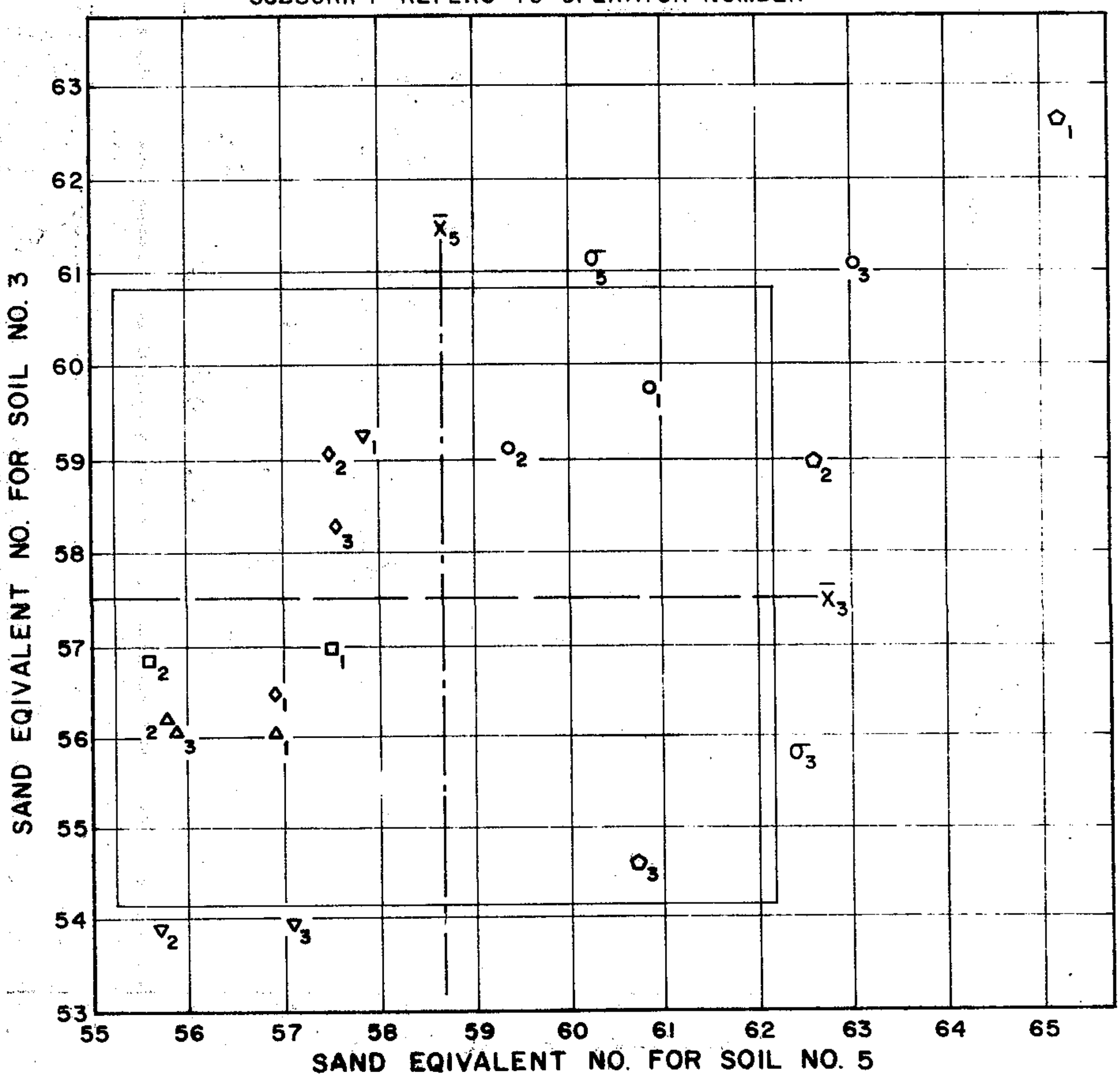


Figure 13

SCATTER DIAGRAM

SOIL NO. 3 VS SOIL NO. 5

IDAHO METHOD

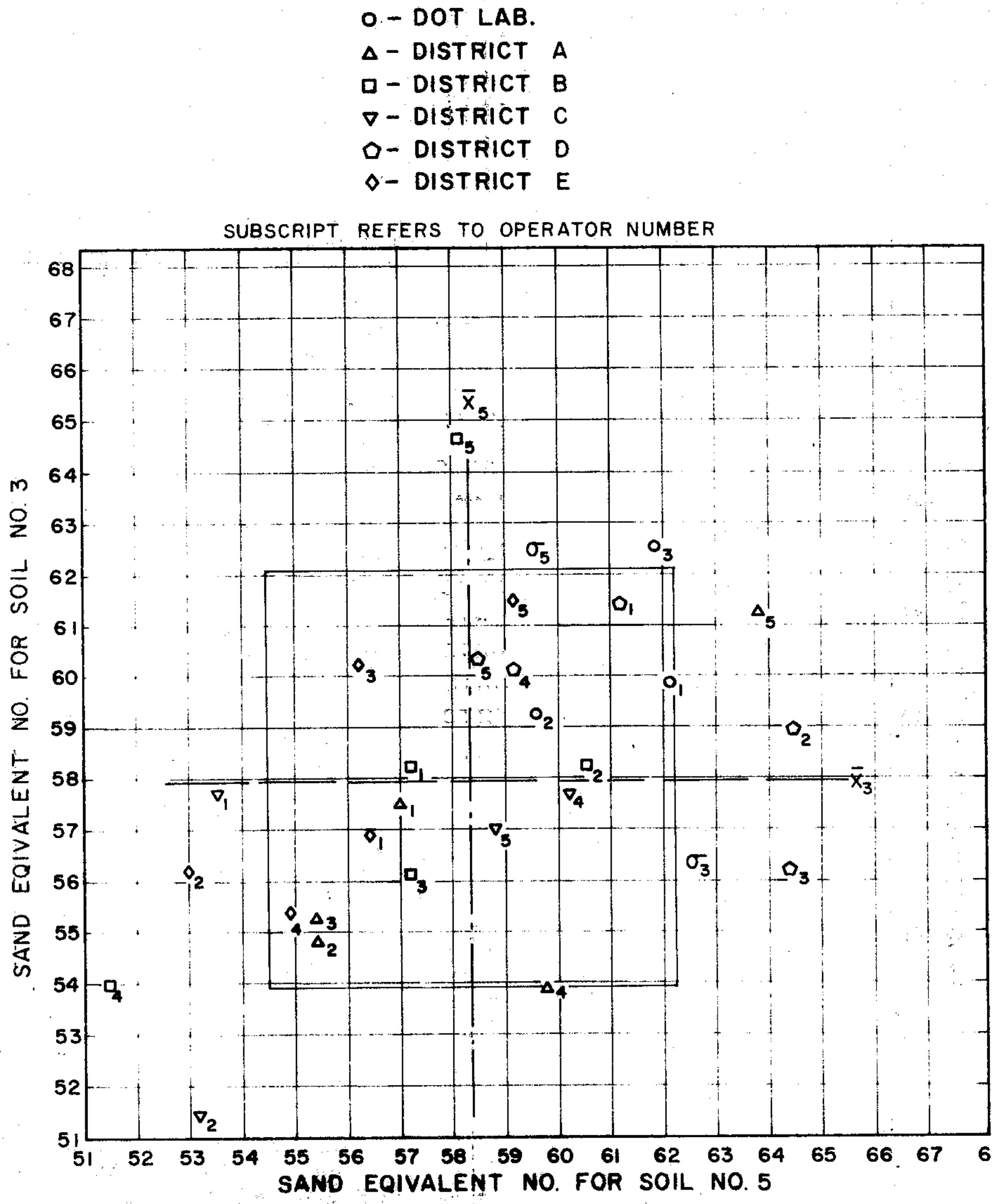
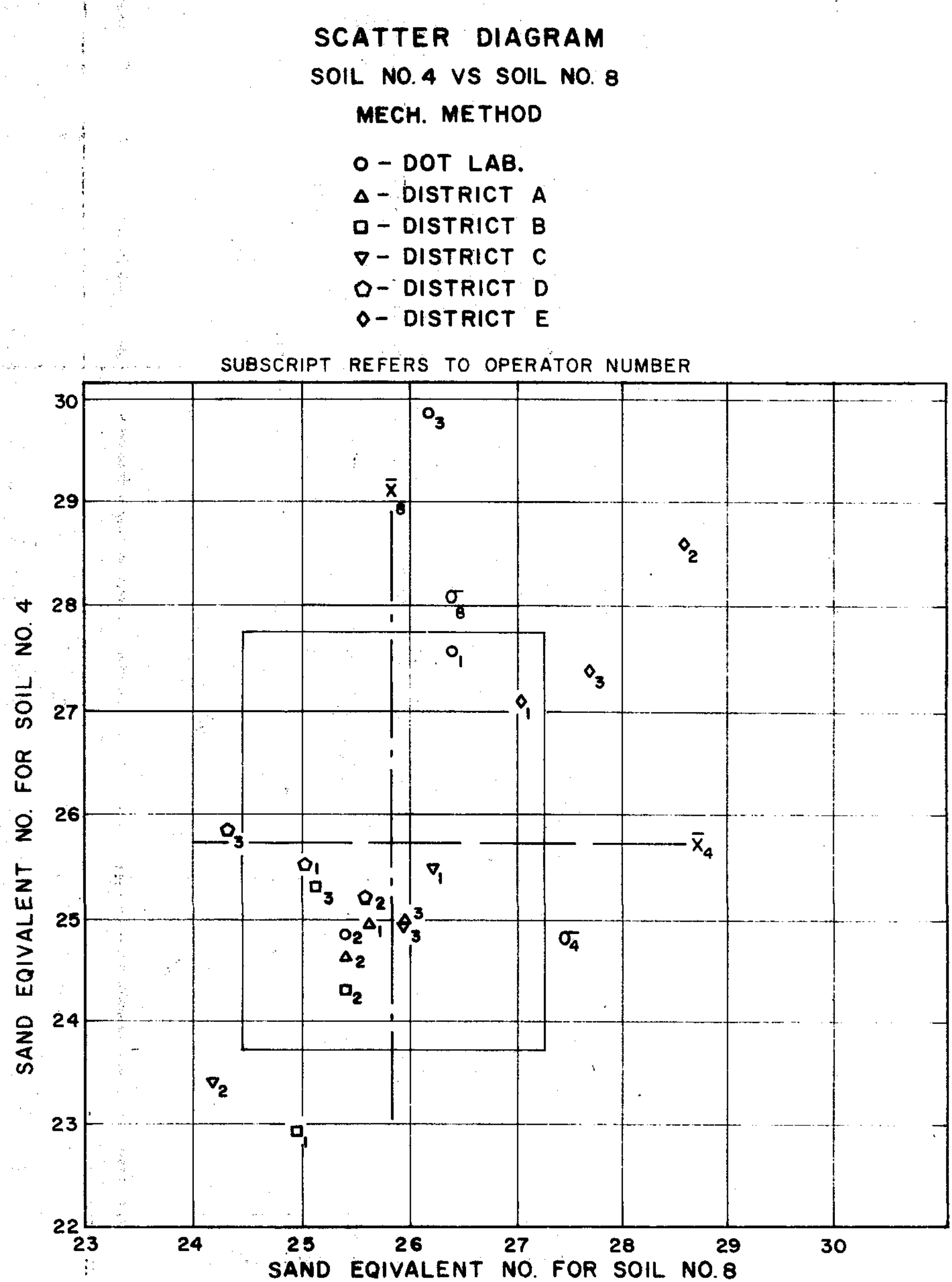


Figure 14



SCATTER DIAGRAM
SOIL NO. 4 VS SOIL NO. 8
MECH. METHOD
○ – DOT LAB.
△ – DISTRICT A
□ – DISTRICT B
▽ – DISTRICT C
⬠ – DISTRICT D
◇ – DISTRICT E
SUBSCRIPT REFERS TO OPERATOR NUMBER
SAND EQIVALENT NO. FOR SOIL NO. 4
SAND EQIVALENT NO. FOR SOIL NO. 8

Figure 15

SCATTER DIAGRAM

SOIL NO. 4 VS SOIL NO. 8

IDAHO METHOD

○ – DOT LAB.
△ – DISTRICT A
□ – DISTRICT B
▽ – DISTRICT C
⬠ – DISTRICT D
◇ – DISTRICT E

SUBSCRIPT REFERS TO OPERATOR NUMBER